V. *Experimental Determination of the Velocity of White and of Coloured Light.*

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PART I.

Absolute Velocity of White Light.

Introductory.

It is remarkable that, although the importance of an accurate knowledge of the velocity of light has been very generally appreciated, no attempt has hitherto been made in this country to measure that velocity by experiment. Our own experiments date from many years back, but we have been prevented by various interruptions to our work from giving a result which could lay claim to the greatest accuracy. In 1878 we made at Pitlochry, in Perthshire, between 600 and 700 observations, but the toothed wheel which was made for us not having the number of teeth in it which we had ordered, we were not able to eliminate perfectly certain unknown quantities occurring in the formulæ, and we felt that it would be better to wait until we could give a result in which we had perfect confidence. At the same time we resolved so to alter our apparatus that we should not have to depend upon the mean of a very large number of experiments to give us a good result, but that each observation should give us an accurate measurement, free from all doubt. This has now been accomplished by the experiments conducted in 1880–81 between Kelly House, Wemyss Bay, and the hills behind Innellan, across the mouth of the River Clyde.

The chief importance of a determination of the velocity of light is that it gives us the means, considered by many to be the best means, of determining the solar parallax, by combining the result with the constant of aberration determined by astronomers. The investigation has also acquired a further interest from the speculations of the late Professor CLERK MAXWELL, according to which the propagation of light is an electro-magnetic phenomenon, and its velocity should be the same as that of the propagation of an electro-magnetic displacement.

Our researches have, however, led us to results of great importance in another direction. We find reasons for believing that the different colours of which white light is composed do not travel with the same velocity, but that the more refrangible rays travel more rapidly through a vacuum, and that this difference is by no means very small, so that we may expect its presence to be determined by independent tests. The influence of this conclusion upon our views about the constitution of the ether is considerable, and will doubtless lead to some further knowledge of its properties.

The general theory of the method employed in the present research resembles that of M. FIZEAU. In his experiments, and in those of M. CORNU, a beam of light is sent through between the teeth of a toothed wheel to a distance of some miles, whence it is reflected back again by means of certain optical arrangements, which bring it back to the same point. If the wheel be rotating at a suitable rate, then, by the time that the light has gone to the distant station and back, the position of the space between two teeth is now occupied by a tooth and the light cannot pass. But if the velocity of the toothed wheel be doubled that position will be occupied by the next space and the light is able to pass. If the velocity be increased threefold, fourfold, &c., we have alternately eclipses and full brightness.

If m be the number of teeth in the wheel, it must, in the time that light has gone to the distant station and back again, have completed $\frac{1}{2m}$ of a revolution in order to bring a tooth into the position previously occupied by a space at the commencement of that time. If the wheel be found to be making N revolutions a second at the time of the first eclipse, then $\frac{1}{2m\mathrm{N}}$ is the fraction of a second taken by light to perform the double journey; and if the distance between the toothed wheel and the distant reflector be D then the velocity of light is

$$\mathrm{V}=4m\mathrm{ND}.$$

In the method which we devised, instead of having only one distant reflector, we have two, nearly in the same line, but one of them being at a greater distance than the other and a little to one side of it. Let us call the most distant reflector A and the other one B. The light reflected from A is eclipsed with a slower revolution of the toothed wheel than that from B; because the number of revolutions required is N, and we have

$$N=\frac{V}{4mD}.$$

But D_A (the distance to A) is greater than D_B (the distance to B); hence N_A (the speed of revolution producing the first eclipse with A) is less than N_B (the speed of revolution producing the first eclipse of B).

After the light from A has been eclipsed it begins to increase in brightness, while that from B is still diminishing. In the method of the present research we determine the speed of revolution when the two lights appear to be of equal brightness.

When we proceed to the second, third, &c., eclipses the difference in speed required to produce an eclipse in A and in B increases, and it may happen that at a certain speed the light from A reaches a maximum at the time when that from B is at a minimum, or *vice versâ*.

The superiority of this method over that of M. FIZEAU seemed to be that instead of having to determine the instant at which a light disappears we have only to determine the instant at which two lights seem to be of equal brightness. Every one who has been engaged in photometry is aware that the former is an operation of great difficulty and doubt, whereas the latter is one of very great delicacy.

The particular way in which we deduce the velocity of light from an observation of the equality of these two lights will now be explained. It must be especially noticed that we never have occasion to use the lights when near an eclipse, at which time, as CORNU has shown, irregularities are introduced into the formulæ.

We found that a great simplification was introduced in the formulæ by observing the 12th and 13th equalities (the ratio D_B to D_A being that of 12 to 13); so that two observations, one at each of these equalities, sufficed to give us a value of the velocity of light. We were also able to utilise some pairs of observations at the 13th and 14th equalities.

This method enabled us to use the electric light, which was in many ways the most convenient to us; for we are not dependent upon the absolute brightness of the light (which of course is liable to variations) but only on the proportionate brightness of the two reflected stars; and to prevent any error arising from any variation in this proportion (owing to fog, &c.), we never used a pair of observations which were separated in time by more than a few minutes.

Mathematical theory of our method.

If k be the width of a tooth, and $1-k$ be the width of the space between two teeth, and if E be the brightness of the star of light seen by reflection from the distant collimator when the toothed wheel is not in position, then the brightness of that star when the toothed wheel is in position and revolving slowly, but fast enough to cause the persistence of visual impressions to prevent us from distinguishing the separate teeth, is $E(1-k)$.

When the speed is increased the intensity depends upon the time taken by light to go to the distant reflector and to return. As the speed of the toothed wheel is gradually increased the star of light diminishes and increases in regular phases.

If the teeth be wider than the spaces the maximum light is less than $\frac{1}{2}E$, and the light is eclipsed not only at certain critical speeds, but also during a change of speed of rotation, which may be considerable if k be much greater than $\frac{1}{2}$.

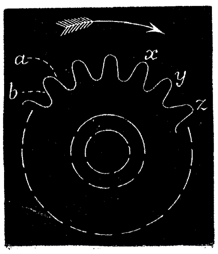
If the spaces be wider than the teeth the maximum light is greater than $\frac{1}{2}E$, and the light is at no speed completely eclipsed, the minimum light being considerable if k be much less than $\frac{1}{2}$.

If the teeth of the wheel be not perfectly uniform, and if some teeth are wider than others, this will alter the law according to which the intensity varies with the speed of rotation. But, if the inequalities be not very great indeed, it is only in the neighbourhood of the maxima and minima that its influence is shown. The general effect is to increase the intensity of light at the minimum and to diminish it at the maximum.

If there be no very great irregularities in the width of the teeth, then (1) the *diminution* of intensity of the light with a definite increase of speed in the odd phases is perfectly constant, provided that in the interval of time taken by light to perform the double journey, the advancing part of each tooth of the wheel passes beyond the advancing part of the r^{th} tooth in front of it, and does not reach the following part of the $\overline{r+1}|^{th}$ tooth nor its following part reach the advancing part of the r^{th} tooth in front, where r is any whole number from zero upwards.*

Similarly, if there be no very great irregularities in the width of the teeth, then (2) the *increase* of intensity of the light with a definite increase of speed in the even phases is perfectly constant, provided that in the interval of time taken by light to perform the double journey the advancing part of each tooth in the wheel passes beyond the following part of the r^{th} tooth in front of it, and does not reach the advancing part of that tooth. This is in the condition where the width of a tooth is less than that of a space. If the width of a tooth be greater than that of a space

* We mean, by this notation, that if in the figure the arrow represents the direction of rotation of the toothed wheel *a* is the advancing and *f* the following part of that tooth, *x*, *y*, and *z* are the third, fourth, and fifth teeth in front of it.



the condition is that in the same interval of time the following part of each tooth should pass beyond the advancing part of the r^{th} tooth beyond it, but should not reach the following part of the $\overline{r+1}|^{th}$ tooth.

It is seen then that with a wheel which is carefully cut the rate of change of intensity of the light with change of speed of the toothed wheel is quite constant at that part of a phase which is about half way between the maximum and minimum brightness.

Our method of experimenting has reference to this part of a phase alone.

Let us call the period,—

From original brightness to the first extinction, the first phase;

From first extinction to next full brightness, the second phase;

From second full brightness to second extinction, the third phase;

and so on.

Let N be the number of revolutions per second made by the wheel when the first central eclipse is attained.

Let m be the number of teeth in the wheel.

Let D be the distance from the toothed wheel to the distant reflector.

Let I be the intensity of the star of light when the number of revolutions a second made by the toothed wheel is n.

Let V be the velocity of light.

Let τ be the time taken by light to perform the double journey, $\tau=\frac{2D}{V}$.

1. In the case of a wheel with teeth of the same width as the spaces (*i.e.*, $\kappa=\frac{1}{2}$), $\frac{dI}{dn}$ is constant, then

$$\frac{dI}{dn}=a;\ I=an+C$$

But if we be considering the first phase, then when $n=0$, $I=\frac{1}{2}E$; and when $n=N$, $I=0$.

Therefore

$$C=\tfrac{1}{2}E,\ a=-\tfrac{1}{2}\frac{E}{N},$$

and

$$I=-\tfrac{1}{2}\frac{E}{N}n+\tfrac{1}{2}E$$

$$\frac{dI}{dn}=-\tfrac{1}{2}\frac{E}{N}$$

2. In the case of a wheel with any width of teeth, the value of $\frac{dI}{dn}$, in the middle of an odd phase, is the same as in the last example, but the maximum intensity is $E(1-\kappa)$. Suppose that it is the p^{th} phase which we are considering, p being odd; then

$$\frac{dI}{dn}=-\tfrac{1}{2}\frac{E}{N}$$

$$I=-\tfrac{1}{2}\frac{E}{N}(n+b)$$

but when

$$n=(p-1)N,\ I=E(1-\kappa)$$

whence

$$b=a-N\{2(1-\kappa)+p-1\}$$

$$I=\frac{E}{2}\left\{2(1-\kappa)+p-1-\frac{n}{N}\right\}$$

3. In the case of a wheel with any width of teeth, the value of $\frac{dI}{dn}$, in the middle of an even phase, is the same as in the two previous examples in magnitude, but of opposite sign, and the maximum intensity is $E(1-\kappa)$. Suppose that it is the p^{th} phase which we are considering, p being even; then

$$\frac{dI}{dn}=+\tfrac{1}{2}\frac{E}{N}$$

$$I=\tfrac{1}{2}\frac{E}{N}(n+c)$$

but when

$$n=pN,\ I=E(1-\kappa)$$

whence

$$c=N\{2(1-\kappa)-p\}$$

and

$$I=\frac{E}{2}\left\{2(1-\kappa)-p+\frac{n}{N}\right\}$$

If we have two distant reflectors, A and B, nearly in the same line, but at different distances D_A and D_B, and having consequently different speeds N_A and N_B, and different intrinsic brightnesses E_A and E_B. Then, if D_A be greater than D_B, N_A is less than N_B, and we shall have equalities of brightness of A and B at different speeds. The first equality is in the first phase of B and the second of A. The second equality is in the second phase of B and the third of A, and so on. The r^{th} equality is in the r^{th} phase of B and the $\overline{r+1}|^{th}$ of A, provided that $r(N_B-N_A)$ is less than $2N_A$. We need not consider any other case.

4. Let us consider the case of the r^{th} equality when r is even. Here B is increasing, A is decreasing. We have then $p_A=r+1$, and $p_B=r$,

$$I_A=I_B;\ \text{or}\frac{E_A}{2}\left\{2(1-\kappa)+(r+1)-1-\frac{n}{N_A}\right\}=\frac{E_B}{2}\left\{2(1-\kappa')-r+\frac{n}{N_B}\right\}^*$$

* We are taking account of the possibility of κ being different for A and B. But in practice this is never the case.

If $E_B = \rho E_A$ this becomes

$$2(1-\kappa)+r-\frac{n}{N_A}=\rho\left\{2(1-\kappa')-r+\frac{n}{N_B}\right\}$$

If ρ, κ and κ' were known we could determine the values of N_A and N_B from this equation when the velocity n giving equality of brightness had been determined, and when the distances D_A and D_B were known $\left(\text{for } \frac{N_A}{N_B}=\frac{D_B}{D_A}\right)$. But these quantities, ρ, κ, and κ', are not known.

5. Let us now consider the $\overline{r+1}|^{\text{th}}$ equality, when r has the same value as before. Here B is decreasing, A is increasing. We have $p_A=r+2$, $p_B=r+1$.

$$I'_A=I'_B\ ;\ \text{or}\ \frac{E'_A}{2}\left\{2(1-\kappa)-(r+2)+\frac{n'}{N_A}\right\}=\frac{E'_B}{2}\left\{2(1-\kappa')+(r+1)-1-\frac{n'}{N_B}\right\}$$

Since E'_B still $=\rho E'_A$, this becomes

$$2(1-\kappa)-(r+2)+\frac{n'}{N_A}=\rho\left\{2(1-\kappa')+r-\frac{n'}{N_B}\right\}$$

6. From these two measurements, viz., of n and n', we can determine N_A and N_B in special circumstances. Let $\frac{D_A}{D_B}=g$. Then we have two equations

$$2(1-\kappa)+r-\frac{n}{N_A}=\rho\left\{2(1-\kappa')-r+\frac{n}{gN_A}\right\}$$

$$2(1-\kappa)-(r+2)+\frac{n'}{N_A}=\rho\left\{2(1-\kappa')+r-\frac{n'}{gN_A}\right\}$$

Subtracting, we have

$$2(r+1+\rho r)=\frac{1}{N_A}\left(\frac{\rho}{g}+1\right)(n+n')=\frac{g+\rho}{N_B}(n+n')$$

If now the distances D_A and D_B have been carefully chosen so as to make $g=\frac{r+1}{r}$, this equation becomes

$$2r(g+\rho)=\frac{g+\rho}{N_B}(n+n')$$

and

$$N_B=\frac{n+n'}{2r}$$

From this we can deduce the velocity of light. For

$$\tau_B=\frac{2D_B}{V}$$

and also

$$\tau_B=\frac{1}{2mN_B}$$

whence

$$V=4mN_B.D_B$$
$$=\frac{2m(n+n')D_B}{r}$$

7. If, however, $\frac{r+1}{r}$ be not quite exactly equal to g, let

$$\frac{r+1}{r}=g+\delta$$

our equation becomes

$$2r(g+\rho+\delta)=\frac{g+\rho}{N_B}(n+n')$$

and

$$N_B=\frac{n+n'}{2r}\cdot\frac{g+\rho}{g+\rho+\delta}$$
$$=\frac{n+n'}{2r}\left(1-\frac{\delta}{g+\rho}+\frac{\delta^2}{(g+\rho)^2}-\ldots\right)$$

and

$$V=\frac{2m(n+n')D_B}{r}\left(1-\frac{\delta}{g+\rho}+\frac{\delta^2}{(g+\rho)^2}-\ldots\right)$$

By employing the first term alone, in this last factor, we obtain an approximate value of N_B which enables us to calculate the value of ρ on the supposition that $\kappa=\kappa'=\frac{1}{2}$ (which is always very closely arranged so in practice). Thus we find the value of the small correction involved in the second term. The third term can always be neglected in our experiments.

8. To find the approximate value of ρ under these circumstances, we have now the two first equations in § 6, reduced to

$$1+r-\frac{gn}{N}=\rho\left(1-r+\frac{n}{N_B}\right)$$

and

$$1-(r+2)+\frac{gn'}{N_B}=\rho\left(1+r-\frac{n'}{N_B}\right)$$

Adding these we have

$$\frac{g}{N_B}(n'-n)=\rho\left\{2-\frac{1}{N_B}(n'-n)\right\}$$

and

$$\rho=\frac{g(n'-n)}{2N_B-(n'-n)}$$

also

$$g+\rho=\frac{2gN_B}{2N_B-(n'-n)}$$

The smallness of the correction due to the term $\frac{\delta}{g+\rho}$ in our experiments can be seen by examining the details of the observations.

In our experiments $g=\frac{13}{12}$ almost exactly. Most of our measurements were made with the 12th equality. Some however were made with the 13th equality. We will now examine the formula for this case.

9. Let us consider the case of the $\overline{r+2}|^{\text{th}}$ equality, r having the same value as before. Here B is increasing, A is decreasing. We have $p_A=r+3$, $p=r+2$.

$$I''_A=I''_B \text{ or } \frac{E''_A}{2}\left\{2(1-\kappa)+(r+3)-1-\frac{n''}{N_A}\right\}=\frac{E''_B}{2}\left\{2(1-\kappa')-(r+2)+\frac{n''}{N_B}\right\}$$

Since E''_B still $=\rho E''_A$, this becomes

$$2(1-\kappa)+r+2-\frac{n''}{N_A}=\rho\left\{2(1-\kappa')-(r+2)+\frac{n''}{gN_A}\right\}$$

But at the $\overline{r-1}|^{\text{th}}$ equality

$$2(1-\kappa)-(r+2)+\frac{n'}{N_A}=\rho\left\{2(1-\kappa')+r-\frac{n'}{gN_A}\right\}$$

subtracting we have

$$2(r+2)+2\rho(r+1)=\frac{1}{N_A}\left(1+\frac{\rho}{g}\right)(n'+n'')=\frac{g+\rho}{N_B}(n'+n'')$$

If now D_A and D_B be so related that $g=\frac{r+2}{r+1}$, this equation becomes

$$2(r+1)(g+\rho)=\frac{g+\rho}{N_B}(n'+n'')$$

$$N_B=\frac{n'+n''}{2(r+1)} \text{ and } V=\frac{2m(n'+n'')D_B}{r+1}$$

If however $\frac{r+2}{r+1}=g+\delta'$ the true value of N_B is

$$N_B=\frac{n'+n''}{2(r+1)}\left(1-\frac{\delta'}{g+\rho}+\frac{\delta'^2}{(g+\rho)^2}-\ldots\right).$$

To approximate to the value of $g+\rho$ on the assumption that $\kappa=\kappa'=\frac{1}{2}$ we notice that the above equations reduce to

$$1+r+2-\frac{gn''}{N_B}=\rho\left\{1-(r+2)+\frac{n''}{N_B}\right\}$$

$$1-(r+2)+\frac{gn'}{N_B}=\rho\left\{1+r-\frac{n'}{N_B}\right\}$$

adding these we have

$$2-\frac{g}{N_B}(n''-n')=\rho\left(\frac{n''-n'}{N_B}\right)$$

and

$$\rho=\frac{2N_B-g(n''-n')}{n''-n'}$$

also

$$g+\rho=\frac{2N_B}{n''-n'}$$

Thus

$$V=\frac{2m(n'+n'')D_B}{r+1}\left\{1-\frac{\delta'(n''-n')}{2N_B}+\dots\right\}$$

when the 13th and 14th equalities are used.

Description of the apparatus.

The optical arrangements devised by M. FIZEAU were admirably suited to the purpose, and were adopted, with modifications, by M. CORNU.

The telescope of emission, which is also the observing telescope, is pointed towards the distant stations. At its focus is placed the revolving toothed wheel; between that and the eye-piece is a diagonally-inclined piece of unsilvered glass.

The reflector designed by FIZEAU for the distant station may be called a reflecting collimator; it consists of a telescope pointing towards the observing telescope, but, instead of having an eyepiece, it has in its principal focus a silver reflector. The advantages of this arrangement are its extreme simplicity and the facility of directing it—great accuracy of adjustment being unnecessary.

The rays of light coming from the sun, or any source of light,* are concentrated by means of a lens to throw an image of the source of light, after reflection at the diagonal mirror, upon the edge of the toothed wheel. If a tooth be not in the way these rays spread out and fill the object-glass, whence they proceed to the distant reflecting collimator, where they arrive all nearly parallel in direction. There they are caused by the object-glass to throw upon the focal mirror a luminous image of the object-glass of the observing telescope. The rays are then reflected to the object-glass, whence they proceed to the observing telescope and produce at its focus an image of the object-glass of the reflecting collimator. If no tooth be in the way,

* In the experiments of 1880–81 we always used the electric light.

the rays proceed onwards through the eye-piece to the eye of the observer, who sees an illuminated image of the distant object-glass, which from its small size looks like a star of light. If now the toothed wheel revolves very slowly he sees the star eclipsed at intervals by the teeth; but if it be revolving so that at least ten teeth pass in a second, then, owing to the persistence of visual impressions he sees the star as a continuous light upon a brightish field produced by the illuminated rotating teeth. If the speed be further increased the brightness of the star diminishes owing to the light which passed between two teeth in leaving the observing station being partially stopped on its return by the advance of a tooth towards that space. So with increasing speed he sees the star disappear, then re-appear, attain its full brightness, diminish, disappear, reappear, &c., passing through similar phases with perfect regularity.

One thing more is required either in the method of M. FIZEAU or our own, and that is to have the means of determining at any instant the velocity of rotation of the toothed wheel. CORNU was the first to attempt to do this with accuracy: he connected the mechanism of the toothed wheel electrically with a chronograph so as to make a mark every 100 revolutions of the toothed wheel. A clock at the same time marked seconds, and by means of a vibrating spring tenths of a second were marked; while a fourth marker was under the control of the observer, who signalled the instant when he wished the velocity to be determined. CORNU did not attempt to maintain a uniform speed of revolution in the toothed wheel, but was able by means of the chronographic record to tell the velocity and rate of change of velocity at definite times, and hence, by interpolation, the exact velocity at any instant.

The plan of the present research was arranged in 1872, but it was not until 1875 that the apparatus was made. Since then the apparatus has been partially modified in order to overcome the optical and mechanical difficulties which arose in the course of the work. The most important pieces were—(1) the telescope, (2) the reflectors, (3) the revolving toothed wheel, (4) the clock, (5) the chronograph, (6) the dynamo-electric machine, and (7) the lamp.

1. *The telescope.*—This consists of a 5-inch achromatic object-glass of good quality, with a focal length of 7 feet. A BOHNENBERGER'S eye-piece is employed, consisting of an erecting eyepiece with a piece of plain glass in front of the field lens and inclined to the axis of the telescope at an angle of 45°. A lateral hole in the tube of the eye-piece allows the light from a lamp, &c., at the side, to be reflected by this diagonal mirror along the axis of the telescope, and thence to the distant reflectors. The adapter, which connects the eye-piece with the body of the telescope, is a tube whose lower half is cut away so as to allow the revolving mechanism to be placed below in such a way as to bring the upper part of the revolving toothed wheel into the axis of the telescope and exactly in its focus. The light from a lamp or from the sun can be concentrated by a lens so as to throw its image upon the top of the toothed wheel. By looking along the axis of the telescope, from the object-glass, an observer can notice whether any of the light falls upon the inner sides of the telescope, and in this manner

he is able to adjust the direction of the beam of light so as to be central, in which case it must go directly to the distant reflectors whose images lie close together, not $\frac{1}{100}$th of an inch apart, on the teeth of the wheel.

Various optical difficulties presented themselves, and the arrangements were altered at various times. These chiefly consisted in illumination of the field of view, and were successively removed. Four improvements specially deserve notice:—

1. A general and intense illumination of the whole field was soon traced to a reflection from the centre of the object-glass. To obviate this a circle of black velvet about one inch in diameter was fastened to the centre of the object-glass on the inside. This was quite successful.

2. In using powerful lights an intense blaze was reflected from the toothed wheel, which made delicate observations impossible. We first tried smoking the toothed wheel, but not only did it still reflect a good deal of light but the regularity of form of the teeth was impaired; finally we used a highly-polished toothed wheel slightly bevelled, and by tilting the revolving mechanism the light was reflected to the upper parts of the interior of the adapter, which being blackened absorbed the light. This arrangement succeeded admirably, but some experiments were also made with the smoked wheel.

3. The field still had some general illumination owing to reflection from grains of dust, or scratches, or other imperfections in the plane-glass diagonal reflector. We objected to the glass reflector also for another reason: at an angle of 45° a *double layer* of glass reflects only 0·16 of the incident light; on the return of the light from the distant reflector 0·84 of this quantity passes through to the eye of the observer. The whole light obtained from this arrangement is then $0·16 \times 0·84 = 0·136$; but if we could have an arrangement to reflect 0·5 of the light, so that on its return we should see $0·5 \times 0·5 = 0·25$ of the light, we should nearly double the intensity. We accomplished this by substituting for the diagonal glass reflector a silvered reflector with an oval hole whose projection on a plane perpendicular to the axis of the telescope is a circle, and whose dimensions are such as to allow one-half of the light to pass through. The diagonal reflector is three inches from the toothed wheel. This arrangement certainly doubled the light, and owing to the darkness of the field its superiority over the glass reflector was enormous.

4. To prevent the slight illumination still remaining on the toothed wheel from causing inconvenience to the observer, a strip of metal with a series of holes of varying size was sometimes placed in the secondary focus of the eye-piece. It could be easily removed and replaced, so that when everything was prepared without its aid this diaphragm was inserted, using so small a hole as to show only the distant reflectors and two teeth of the wheel.

*The toothed wheel.**—In contriving the revolving toothed wheel and the mechanism

* The term "*the* toothed wheel" applies in this description to that toothed wheel in the mechanism between the teeth of which the light is made to pass.

which gives it motion, considerable forethought is necessary. It must revolve at a very great speed, and must be capable of going at least for some minutes so as to avoid the necessity of continually winding it up. Our apparatus was constructed by Messrs. E. DENT and Co. It is mounted upon three screws, which rest in three holes countersunk in a plate of iron, which latter has an oblong hole in it to admit the passage of the catgut supporting the weight. This base plate of iron is, as before stated, generally slightly inclined by resting on a wedge, so that the light striking the toothed wheel is reflected upwards. Each wheel works into a pinion so as to multiply the velocity tenfold. These wheels and pinions are on five separate arbors, so that the multiplication is altogether 10,000-fold. On the first arbor the drum, 3 inches in diameter, is fixed. To the second a circular disc with a milled edge is attached. This enables the observer to increase or diminish the velocity of the wheel by a touch of the hand. It also enables him to reduce the velocity before putting on the brake, a sudden application of which might injure the mechanism. On the third arbor there is a cam which makes contact between two platinum points supported by two springs once in a revolution (*i.e.*, 100 revolutions of *the* toothed wheel), by means of which a current of electricity is transmitted to the chronographs. On the fourth arbor there is a brake consisting of a light disc of metal caught between two springs. To release this brake a handle or key behind the mechanism is turned through a right angle. This separates the springs and the brake ceases to act. The fifth arbor supports *the* toothed wheel, which is $1\frac{1}{2}$ inch in diameter, and contains 400 teeth cut to a depth of $\frac{1}{16}$th of an inch. The wheel is bevelled so as to reflect the light away from the observer's eye. Teeth of different shapes were tried, but the best was found to be that of saw teeth (*i.e.*, with pointed teeth and pointed spaces). The width of the teeth could then be varied by raising or lowering the mechanism by means of the foot screws. In the same way the toothed wheel could be brought exactly into the focus of the telescope. Wheels with varying numbers of teeth were also tried, and that one with 400 teeth was chosen as giving the best results.

The reflectors.—The two reflecting collimators are of identical construction. Those which we found to be most suitable were a pair constructed by Messrs. TROUGHTON and SIMMS. They are supplied with achromatic object-glasses of 3 inches diameter and 3 feet focus. At the other end of the tube a cap is screwed on, and to the centre of this cap a circular silver mirror is attached by three screws which admit of adjustment. This mirror is ground to a spherical form, the radius of the sphere being 3 feet, and the centre of the sphere of which it is a part being the centre of the object glass. Other collimators were tried, in which the object-glasses were replaced by 9-inch silvered glass reflectors of parabolic form. This pair was constructed by a maker who makes a speciality of such reflectors for telescopes; but the whole workmanship was so disgracefully bad in every part, every conceivable fault being found in it, and every known device for patching up and concealing bad work having been resorted to, that in the form supplied to us they were absolutely useless. They were discarded altogether when we found that the atmospheric conditions were seldom favourable enough to admit of our using a larger aperture than 3 inches. We have always used the 3-inch collimators in our observations. Three adjustments must be made: (1), for focus; (2), for centering; (3), for direction.

(1.) To adjust the focus, the cap with the mirror attached is removed and another one put in its place. This one is similar to it in every way except that the mirror is replaced by a piece of ground-glass whose surface has the same position with respect to the cap that the mirror has in the other one. The image of a star or the sun can be thrown upon the ground-glass and focussed. The number of turns and parts of a turn of the screw of the cap are then counted, and the cap with mirror is now screwed on to the same position. A final adjustment can be made with the collimators in position, altering the focus until an observer at the telescope sees the reflected light most distinctly.

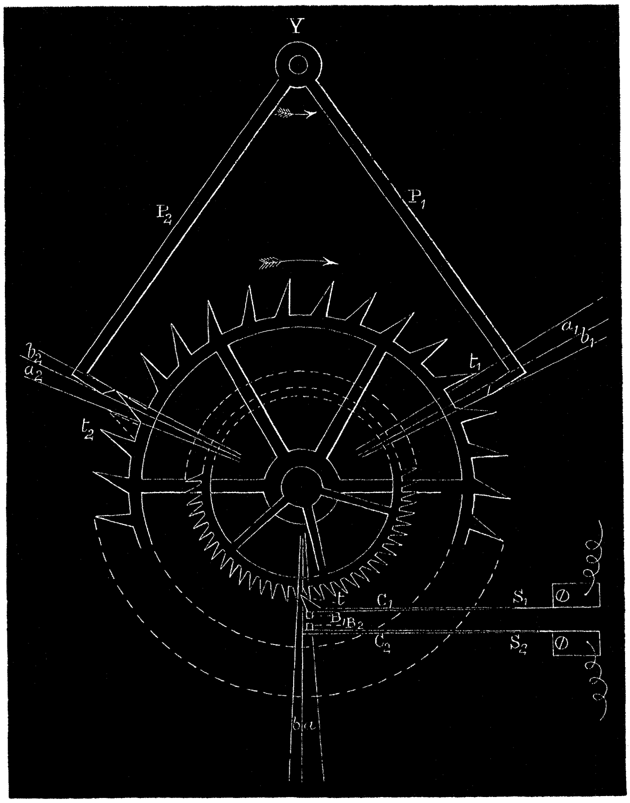
(2.) The adjustment which we call centering is accomplished by the three screws which attach the mirror to the cap, and causes the centre of the sphere of which the mirror forms a part to be at the centre of the object-glass. To test this point a tube 1 foot long is put on the collimator projecting in front of the object-glass. At the end of this tube a small ring is supported in the centre by three strips of metal. On looking through this ring the observer ought to see an image of his eye, and the screws are adjusted until this is the case.

(3.) To direct the collimators so that their axes should point to the observing telescope it was found most convenient to look through the object-glass at the mirror. If the head be so far as possible prevented from covering the object-glass an image of distant objects is seen in the mirror, and the direction is changed until the light coming from the observing telescope is seen in the centre of the mirror.

The clock.—The clock was constructed by Messrs. E. DENT and Co. specially for this research. It is driven by a weight attached to an endless chain passing over the drum. On the arbor of the drum, and connected with it by a toothed wheel and ratchet, is a wheel of 100 teeth working into a pinion of 10 teeth, which is on the same arbor as the scape wheel. This arbor bears a hand which marks seconds, and the arbour of the large wheel bears a hand which marks the minutes from 1 to 10. Any additional wheelwork was unnecessary for our purpose and would only tend to introduce errors. There is a dead-beat escapement, and the pendulum rod is made of varnished wood. The bob weighs 6 lbs., and the driving weight only 4 ozs., or one-twenty-fourth of the weight of the bob. The clock is best wound up by unhooking the weight from the endless chain after having hooked on another higher up. It goes without attention for two hours.

The arbor of the scape wheel bears another similar one, with 120 teeth, for making electrical contact once a second, by which means a mark shall be made on the chronograph. The system of electrical contacts adopted by us seemed to be free from the defects of those which tend to disturb the pendulum, because the springs whose contact sends the electric current are moved during the drop of the scape wheel from one pallet to the other. In the figure (fig. 1) a tooth has just passed from the impulse face of the pallet P_2. t_1 now falls through the angle a_1 on to the dead face of the pallet P_1. *During this interval* the tooth t of the smaller* wheel mounted on the same axis as

Fig. 1.



the scape wheel moves the contact spring C_1 S_1 against the contact spring C_2 S_2, and the current passes. (The point of C_1 is agate, and the points of contact are platinum.) It will be seen that when the tooth t_1 goes on to give impulse to the pallet P_1 through the angle b_1 the tooth t has allowed the springs to separate and it passes through its angle b undisturbed. The motion of the tooth t_2 can be easily traced from the drawing. Careful workmanship was required to carry out this plan, as the angles a, a_1, a_2 ought to be made smaller than in the drawing. The result has been very successful.

Nevertheless, although this arrangement seems admirably adapted for not interfering with the regularity of the pendulum's vibration, we considered that it was not perfectly satisfactory for our purpose. For the length of a second marked by it on the chronograph would depend upon the exact form and size of the teeth in the auxiliary wheel. Consequently, we extemporised a contact free from this defect. A hole of the same size as the arbor of the crutch was bored in a piece of cork, and

* The wheel is here drawn smaller for clearness of description. It is really of the same size.

a slit was made in the cork to allow it to be fixed upon that arbor. Through the cork a thick copper wire was passed stretching horizontally for 3 inches, where it was bent downwards at a right angle. This bent arm was half an inch long, was amalgamated, and dipped, at each oscillation of the pendulum, into a cup of mercury connected with the electric circuit. The stout copper wire was also connected with that circuit by means of a very fine copper wire. This arrangement was very perfect.

The chronograph.—In all the experiments previous to 1880 we used one of the ingenious portable chronographs constructed by M. HYPP, of Neuchatel. In this instrument a strip of paper is run off from a drum, and upon it the signals from the clock and wheel are inscribed by two siphon pens. Uniformity of motion is given by the mechanism of the chronograph giving motion to a spur wheel against which the end of a spring presses. This spring permits one, two, &c., teeth of the wheel to pass in the course of a single vibration, the number allowed to pass being under control of the experimenter. A large number of observations were made with this apparatus. But we came to the decided opinion that while it is admirably adapted for observatory work where a greater accuracy than $\frac{1}{100}$th second is not required, yet in order to get the most perfect possible results a different class of instrument must be employed. Consequently we designed a new chronograph, which was constructed for us by Messrs. ELLIOTT Brothers. The principle of this apparatus is that we depend for uniformity of motion on the inertia of the apparatus. It was our object to get rid of all clockwork, and by making use of a fly-wheel, which has no work to do, to get rid of a host of irregularities which affect (it may be in a very slight degree) all other chronographs.

The base of the instrument consists of two strong triangular castings bound together in a horizontal position by three brass pillars. Between these triangles a fly-wheel, 12 inches in diameter, rotates on a vertical axis. The lower end of this axis is a hard steel socket which rests upon the point of a strong screw working in the centre of the lower triangle and firmly fixed there by means of a nut.

The lower triangle is supported upon three feet adjustable for levelling by screws. To the upper triangle two vertical brass pillars, 16 inches long, are fixed. They are connected at the top by a cross piece of iron, in the middle of which a thick pin works vertically with a screw motion, and it can be fixed by a nut.

The lower end of this pin terminates in a point, supporting the hard steel socket at the upper part of the axis of the registering cylinder. This cylinder is of brass. It is 12 inches long and 4 inches diameter. Its axis projects an inch at each end. The upper end, as before said, rests against the point of the upper screw. The lower end is, during an observation, firmly fixed to the axis of the fly-wheel by means of a solid brass tube closely fitting these two axes, with six screw nails to ensure rigidity. Round the upper end of the axis a silk thread is wound which, passing over a pulley mounted on friction wheels, carries a small weight. The axis of the cylinder is made vertical by the levelling screws, a spirit-level being placed on the top of the cylinder and observed in different positions of the fly-wheel. Thus we have a rigid mass rotating about a vertical axis with no cause for irregular motion except the acceleration produced by the weight and a slight friction, which combine to produce an accelerating or retarding force which may be regarded as constant. Two cylinders, exactly similar, were employed, and these could be interchanged. The brass cylinders were covered with a thin layer of smoke from a paraffin lamp.

The marks made by the currents of electricity from the clock and wheel respectively, were produced by two aluminium points attached to two springs, which were attracted by the poles of two electro-magnets in the circuits of the clock and wheel respectively. Thus, while the pens are in contact with the cylinder if there be no currents passing, and the fly-wheel be rotated, two circles are traced upon the cylinder. The electro-magnets are mounted on a stand which slides up and down one of the vertical brass pillars by means of a rack and pinion. Thus the assistant can, by turning the pinion, convert the two circular marks into spirals. We judged that it was much better to make an assistant perform this work than to use up the energy of the fly-wheel in the same way, as is often done in chronographs. Moreover, he turns the pinion only while an observation is being recorded, and not in every case when the fly-wheel is rotating, thus effecting a great saving in the space on the cylinder. The electro-magnets are not fixed rigidly to this support which moves up and down on the vertical column. They are attached to it by a vertical axis about which they have a small motion. Thus we are able to keep the pens in contact with the cylinder, or not, as we please. Ordinarily the pens are kept away from the cylinder by a light spring. By sending a current of electricity through a third electro-magnet, the pens are brought into contact with the cylinder. This current is sent by the observer by means of a contact maker, a few seconds before making an observation. When the assistant sees this he moves the pinion so as to cause the pens to describe a spiral, and the observer, at the instant of an observation, breaks and makes contact rapidly. This leaves a break on the smoked cylinder for about one-eighth of a second, the beginning of the break indicating the time of the observation.

When it is desired to read off the records of observations the cylinder is first taken off, and the collar which connected it rigidly to the fly-wheel removed. A ring, to which an arm carrying a vernier is attached, is now screwed to the centre of the upper triangle, and forms a bearing in which the axis of the fly-wheel can work. This part of the axis is conical, having the smallest diameter above. On this conical pivot a conical collar, which carries a 5-inch divided circle, is jammed. The cylinder is replaced with the lower end of its axis resting on the pointed upper end of the axis of the fly-wheel. It is prevented from rotating independently of the fly-wheel by an upright piece of metal and an upright spring, which are attached to the divided circle, and which catch between them one of four radial arms at the base of the cylinder.

It has been stated that one of the vertical pillars carries the marking pens or styles. The other pillar bears an arm to which is attached a microscope with cross wires in its focus. This microscope can be raised or lowered on the pillar which supports it. On looking through this microscope the marks made by one of the pens can be seen and brought, by rotating the fly-wheel, into super-position on the cross wires. The position of the cylinder is then read off by the vernier. The circle is divided into 500 equal parts, and by aid of the vernier we can read to one-twentieth part of one of these divisions, *i.e.*, to $\frac{1}{10000}$th of a revolution. We found it best in practice to rotate the apparatus at the rate of about one revolution a second. Hence one division of the vernier corresponds to about $\frac{1}{10000}$th of a second.

The perfect working of the chronograph depends upon a number of conditions which must all be fulfilled simultaneously. They are dependent (1) upon the manufacture of the instrument, (2) upon the mechanical adjustments, and (3) upon the electrical adjustments. The two last alone can be improved by the observer. The principal objects to be sought after in the manufacture of the instrument are absence of friction, rigidity, and the reduction of the moment of the momentum of the fly-wheel and cylinder together about their axis to zero.* The mechanical adjustments are as follows :—The vertical position of the axis of the instrument must be carefully tested by a level. The points of support must be oiled. The screws connecting the cylinder to the fly-wheel must be quite tight. The pressure of the upper point of support upon the axis must be very nicely adjusted. This point of support must be kept rigidly in position by the nut. The electrical adjustments are as follows :—The batteries and magnets must be in good working order. The pens must move freely and over a suitable range, and they must press with a suitable pressure upon the smoked cylinder.

The dynamo-electric machine.—The electric current to work our electric lamp was obtained from a small sized SIEMENS' dynamo-electric machine requiring three horse-power and rotating at the rate of 1,400 turns a minute. The axis of the machine is attached directly to a turbine of that kind known by the name of a vortex turbine. This was fed by a supply of water with a head of 300 feet coming from a distance of about a mile, and led to the house by a 3-inch water pipe. The current was led by means of stout wires from the generator to the lamp, a distance of about 100 feet. No further account of this machine is necessary, as our practice only, and not our results, could be affected by any imperfections.

The source of light.—We employed for our source of light, in the experiments of 1880–81, a SIEMENS' electric lamp. A condensing lens was used to throw an image of the incandescent carbon, after reflection by the diagonal reflector, upon the toothed wheel. Since with this lamp motion is given to only one of the carbon points it was necessary to mount the condensing lens upon a framework which should admit of its having an up and down movement. For the same reason the diagonal reflector in the eye-piece of the telescope was turned round about the axis of the telescope at each

* The maker did not quite fully carry out our ideas on the two last of these points. The rigidity of the apparatus would have been increased by having three vertical pillars in place of two.

observation so as to send a maximum quantity of light along the axis to the distant reflectors. In some of our later experiments we used a bisulphide of carbon bottle-prism to send particular colours to the distant reflector. This prism was laid upon a ledge on the support of the condensing lens and immediately below it. With a little care a very pure spectrum could thus be formed.

Establishment of the apparatus at Kelly, Wemyss Bay.

The room in Kelly House (the property of Dr. YOUNG) which was used for the observatory was the billiard room, facing the front of the house (the west). It is the room immediately to the north of the entrance hall or lobby. A pane of glass was removed from the window for observation. On the window-sill a brass plate has been fixed, with an inscription, indicating the exact relative position of the toothed wheel. The telescope was mounted upon two strong wooden supports. The toothed wheel mechanism rested upon two very solid beams (between which the cord for the weights could pass). These beams rested at one end upon a standard fixed to the wall of the room, and at the other end upon a strong box which rested on the billiard table. This end was also supported by a standard; and the floor was strengthened by thick wooden props below. The horizontal beams also acted as a support for the framework bearing the condensing lens and prism. The electric lamp was on a separate table. At one end of the beams and at the observer's right hand mercury cups were placed, with metallic contact pieces, by means of which the batteries could be put out of action when not required. LECLANCHÉ cells were used for the chronograph connexions. Twelve cells were used in all. The clock was attached to the wall of the room facing the observer. The levelling screws of the chronograph rested upon three metal plates on a piece of wood on the billiard table.

We have inserted a brass plate in the stone outside the window, with an arrow pointing towards the position occupied by the toothed wheel, the distance from the point being 106 inches.

Establishment of the reflecting collimators.—The reflecting collimators are placed on the hills behind the village of Innellan, separated from the observing telescope by the Firth of Clyde. The nearest one (B) is on the summit of the hill called *the Tom.** It rests upon two iron Y's which are imbedded in the solid rock. The reflector was 8 inches further from Kelly (*i.e.*, more to the west) than the most westerly of these two Y's, whose positions will always be marked by the holes. The collimator was covered by a wooden box with a hole in the east side. This box was attached to four iron rods placed in holes in the rock.

The more distant reflector (A) is in the face of the hill to the west of the Tom, nearly in the same line as a line drawn from Kelly to the top of the Tom, but two feet to the north of this line. It was impossible to gain a solid foundation, but the

* Kelly House and *the Tom* are both shown on the Ordnance Survey Maps.

face of the hill was partially excavated, and two holes were "jumped" in a massive stone, in which two Y's were fixed to support the reflecting collimator. The reflector A was $8\frac{3}{4}$ inches further from Kelly than the most westerly of these two holes.

The reflector A was frequently shifted in direction (not in absolute position) by the unsatisfactory condition of the soil. This was especially the case during the severe frost of the winter, and on the occasion of the subsequent thaw. It was covered by a wooden box, open to the east.

The reflector A was not interfered with by any inquisitive persons. But the box covering B was once broken open at the west end, and the centreing screws abstracted, doubtless with the aim of obtaining a view of Kelly House with what was supposed to be a powerful telescope.

We made some sketches, and took some photographs of the apparatus, from which the accompanying plates have been prepared.

[*Note.*—It has been decided that the descriptions of the apparatus are sufficiently clear, and that it is unnecessary to supplement them by the plates to which allusion is here made.]

The general arrangement of our apparatus gave us considerable satisfaction. We could have wished, however, to have had a chronograph of the same kind, but more convenient in use in some ways, with greater rigidity, and with a smaller moment of momentum about the axis. The mechanism of the toothed wheel might perhaps be constructed so as to give a greater velocity than we were able to obtain. If a better climate were experienced considerable advantage would be gained, and the telescope and reflectors might then be increased in size, and the distances lengthened.*

†[*Measurement of the distances between the toothed wheel and the reflecting collimators.*

These measurements were made with the ordnance survey 20-inch theodolite. The base-line was the distance between the ordnance survey centre-marks on Knock Hill and Innellan Hill. The necessary information was furnished us by Colonel CLARKE. It seems unnecessary to give details of the triangulation, the manuscript being left in the charge of the Royal Society for reference. Calling Kelly House Station C, and the reflectors Stations A and B respectively, we obtained

CA=18,210·6 feet
CB=16,825·3 feet.]

These distances require several corrections.

* We wish to record our indebtedness during the whole series of the observations which follow, to the able and skilful assistance rendered to us by Mr. D. STEWART, who superintended the working of the chronograph, and aided us in the work generally.

† Details of triangulation have been left out. The part in square brackets here is substituted.—G. F., December 17, 1881.

(1.) The distances of the reflectors and toothed wheel form the points used in the triangulation. These are

at A—9·18 feet

at B—0·97 foot

at C+8·83 feet

giving a correction to

CA=—0·35 foot

and to

CB=+7·86 feet.

(2.) The thickness of the object-glasses, giving to CA and to CB a correction of +0·1 feet to each.

These two corrections make CA=18,210·3 feet and CB=16,833·3 feet.

(3.) The reduction to a sphere whose radius is the distance of Kelly from the earth's centre instead of the sea level. The height of Kelly above the sea level is 100 feet. This correction is quite insensible.

(4.) The reduction of the circular measure of the angle subtended by CA or CB at the earth's centre to the chord of that arc. The angle is about 3′, and the reduction is quite insensible.

(5.) The station B is 414 feet higher than Kelly, and A is also in the same line. Owing to this cause there is a correction of +1·7 feet to CB and +1·9 feet to CA.

(6.) Lastly, the rays of light do not go in a straight line from C to A or B because they are bent into a curve by refraction. The curvature is so small that we may consider it as a part of a circle of very large radius. Supposing that the refraction amounted to half as much as the horizontal refraction of a heavenly body, *i.e.*, to 15′, then the angle subtended by this part of a circle would be 30′. But the difference between the circular measure of 30′ and the chord of the arc is insensible. Hence this correction is insensible.

The final result is that the distance CA=18,212·2 feet and CB=16,835·0 feet. Or D_A=CA=3·44,928 miles. D_B=CB=3·18,845 miles.

$$g=\frac{D_A}{D_B}=1·08181$$

$$\delta=\frac{13}{12}-\frac{D_A}{D_B}=+0·00152$$

$$\frac{r+1}{r}=\frac{13}{12}=1·08333$$

$$\delta'=\frac{14}{13}-\frac{D_A}{D_B}=-0·00484$$

$$\frac{r+2}{r+1}=\frac{14}{13}=1·07697$$

Translation of the chronograph records.

The most usual way of observing with the chronograph was to complete the electric circuits for the clock and the rotating mechanism during the hour or two that we might be at work. The circuit which when closed brings the pens into contact with the smoked cylinder was however kept open, the means of closing it being a contact key in the hand of the observer. The assistant winds up the chronograph (which can only go for about 20 seconds) and levels it. He also examines the pens to see that they are in working order. The observer winds up the mechanism. He notices that his contact maker is in good working order. He oils the pivot holes of the mechanism. He puts on the number of weights which experience tells him will give the required speed approximately. He asks "Are you ready?" When the assistant is ready the observer releases the brake on the mechanism. He may if he please count the number of eclipses of A or B before his final speed is obtained. He adjusts the width of the teeth by raising or lowering the mechanism by means of one of the levelling screws. When the speed which gives equality of lights is nearly obtained he notices and records whether A or B is increasing with increase of speed. He makes the equality of lights more exact by adding to or taking from the driving weights, or by pulling the catgut supporting the weights either up or down, or by touching the milled head on the arbor of the second wheel on the mechanism, or in such a way as he has decided upon, until he feels that he has complete power to produce equality of lights. He then says to the assistant "Begin." The assistant gives a definite speed (judged by the eye) to the fly-wheel and cylinder of about one revolution a second. After a few seconds the observer assumes that the cylinder is rotating uniformly. He makes contact. The pens touch the smoked cylinder. The assistant immediately begins to lower the support of the pens by means of the rack and pinion, so as to make the pens describe spirals close to each other, the pens meanwhile marking seconds of the clock, and hundreds of revolutions of the toothed wheel respectively. The observer now adjusts the speed so as to produce equality of brightness in the two stars. When he is quite certain that he has attained this he breaks contact for one-eighth of a second or thereabouts. This leaves a blank in the spiral traces, the commencement of which indicates the time at which the speed is required.

The 12th and 13th equalities were generally used, and sometimes the 14th. Successive observations were always made at successive equalities to allow of the complete elimination of errors alluded to in the theory of our method.

The cylinder is completely covered after four or five observations have been made.

After the spirals have been traced by the clock-pen and the wheel-pen upon the smoked cylinder of the chronograph, the divided circle and vernier are attached to the instrument, the former being divided into 500 divisions, and the latter reading to $\frac{1}{10000}$th of a revolution. The microscope with its cross wires is placed so that the irregularities marked by the pens on the spiral traces can be readily seen, and brought to coincide in succession with the vertical wire. This done, a reading is taken, and the cylinder is turned round until another mark coincides with the vertical wire. So the operation is continued, first with the wheel marks, and afterwards with the clock marks. In reading off the marks on each side of a break in the traces care must be taken to notice whether a mark (or it may be even two marks) have been omitted. Care must also be taken to notice the number of whole revolutions of the cylinder. But it is only in the case of the clock-pen that this is necessary; and even in this case the omission of this precaution could hardly lead us into error. In this way we may obtain a series of readings for the wheel and clock respectively such as the following:—

January 20, 1881, No. 6.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			515		
231			s_1 9,890	22,557	
	2,411	1,437	23,072	(22,419)	329
2,642			s_2 32,309	22,228	295
..	2,404	5,046	45,300	22,124	
7,449			54,433		
Signal at 3,810			Signal at 24,108		

The signals are indicated by the commencement of the break in the pen traces. Notice that in the reading of the wheel marks the third one is a blank, indicating that one mark is omitted in the break. In consequence of this the corresponding "difference" is half of the difference between the two numbers 2,642 and 7,449. Notice also that in taking the clock differences we must subtract the *alternate* readings, partly because the clock cannot be perfectly "in beat," and partly because the electrical connexion is such as to make the length of the second, as registered, depend upon the direction of vibration of the pendulum.

The "difference," such as 2,411, in the column of the wheel record, is the number of divisions (or ten-thousandths of a revolution) passed over by the cylinder in the course of 100 revolutions of the toothed wheel.

The "alternate difference," such as 22,419 in the column of the clock record, is the number of divisions passed over by the cylinder in the course of two seconds of time.

If the cylinder rotated with perfect uniformity the quotient of these two numbers, viz., $\frac{22419}{2411}=9{\cdot}2986$, would be the number of hundreds of revolutions completed by the toothed wheel in two seconds of time, and if we multiply this number by 50, we get 464·93 revolutions per second as the speed of the toothed wheel.

But as a matter of fact the chronograph-cylinder is constantly accelerated by the small weight, and retarded by the friction of the instrument. The total effect is generally a retardation, gradually diminishing the velocity of rotation of the chronograph. In the short interval of time (usually about one-fourth or one-fifth of a second) between two wheel marks this retardation does not affect us. We can say, in the above example, that the cylinder is passing over 0·2411 revolution in 100 turns of the toothed wheel, at the instant when the pen is at the reading 1,437; and that it is passing over 0·2404 revolution in 100 turns of the toothed wheel when the pen is at the reading 5,046. A simple proportion tells us that when the pen is at the reading of the signal, *i.e.*, 3,810, the number of revolutions in 100 turns of the toothed wheel is $0{\cdot}2411-\frac{2373}{3609}\times 0{\cdot}0007=0{\cdot}2406$.

But in the case of the clock trace we use intervals of two seconds. Here the case is very different. From the readings of the chronograph we must determine what was the actual velocity of the cylinder at some particular reading of the chronograph, what was the retardation produced by the excess of friction, what is the law according to which the velocity of rotation of the cylinder varies with the reading upon the cylinder; and thence what was the velocity of its rotation at the time when the signal was indicated.

Let s be the reading at any time t, and v the velocity of rotation of the cylinder.

When $t=0$, let $s=s_0$, and let $v=u$.

Let $s=s_1$ and s_2 when the times are t_1 t_2 respectively.

Let f be the excess of friction, or the retardation.

Then

$$s_1=s_0+ut_1-\tfrac{1}{2}ft_1^2$$

$$s_2=s_0+ut_2-\tfrac{1}{2}ft_2^2$$

$$s_2-s_1=u(t_2-t_1)-\tfrac{1}{2}f(t_2^2-t_1^2)$$

$$\bar{v}=\frac{s_2-s_1}{t_2-t_1}=u-f\frac{(t_2+t_1)}{2}$$

Therefore $\bar{v}$=the velocity of rotation at the time $\frac{t_2+t_1}{2}$.

Now let $\bar{s}$ be the reading corresponding to the velocity of rotation $\bar{v}$, or the reading at that time $\frac{t_2+t_1}{2}$.

$$\bar{s}=s_0+u\frac{t_2+t_1}{2}-\tfrac{1}{2}f\left(\frac{t_2+t_1}{2}\right)^2$$

but

$$\frac{s_1+s_2}{2}=s_0+u\frac{t_2+t_1}{2}-\tfrac{1}{2}f\left(\frac{t_2^2+t_1^2}{2}\right)$$

Therefore

$$\bar{s}=\frac{s_1+s_2}{2}+\tfrac{1}{8}f(t_2^2+t_1^2-2t_1t_2)$$

$$=\frac{s_1+s_2}{2}+\tfrac{1}{2}f\left(\frac{t_2-t_1}{2}\right)^2$$

In our case $t_2-t_1=2$ seconds. Hence we have

$$\bar{s}=\frac{s_1+s_2}{2}+\tfrac{1}{2}f, \text{ corresponding to } \bar{v}=\frac{s_2-s_1}{2}.$$

To determine the value of f we notice above that

$$s_2-s_1=u(t_2-t_1)-\tfrac{1}{2}f(t_2^2-t_1^2)$$

so

$$s_3-s_2=u(t_3-t_2)-\tfrac{1}{2}f(t_3^2-t_2^2)$$

(where s_3 is the reading of the clock mark two seconds later than the mark indicated by s_2), but $t_2-t_1=t_3-t_2=2$ seconds. Therefore, subtracting, we have

$$(s_3-s_2)-(s_2-s_1)=f(t_3-t_1)$$
$$=4f$$

whence

$$f=\frac{(s_3-s_2)-(s_2-s_1)}{4}$$

It follows then that f is equal to the second alternate difference in the above example divided by 4.

This value of f, thus determined, is never perfectly constant; but we can interpolate so as to find its mean value in the interval between the reading $\frac{s_1+s_2}{2}+\frac{1}{2}f$ and the reading corresponding to the signal given by the observer.

[*N.B.*—When through imperfection of adjustment the values of f were very discordant, the observation was always rejected.]

To deduce the velocity v of rotation of the cylinder when the reading (corresponding to the signal) was s, we notice that

$$v^2=\bar{v}^2-2f(s-\bar{s})$$
$$=\left(\frac{s_2-s_1}{2}\right)^2-2f\left(s-\frac{s_1+s_2}{2}-\tfrac{1}{2}f\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

The first thing to be done in applying these formulæ is to determine the value of f which is to be adopted. The record given above shows us that, at the reading 23,072, $4f=329$; and at the reading 32,309, $4f=295$; and we obtain for the value of $4f$, at the signal reading 24,108, by simple proportion,

$$4f_1=329-\frac{1036}{9237}\times 34=325.$$

Taking this as a first approximation to the value of $4f$ which is to be adopted, we find

$$s_1 = 9,890$$
$$s_2 = 32,309$$
$$s_1 + s_2 = 42,199$$

$\bar{s} = \frac{s_1+s_2}{2} + \frac{1}{2}f = 21,100 + 41 = 21,141$ (assuming f_1 is sufficiently near to the correct value of f).

The value of $4f$ corresponding to this value of s is

$$4f_2 = 329 + \frac{1931}{9237} \times 34 = 336.$$

The value for $4f$ which we must adopt is the mean of these two, $4f_1$ and $4f_2$.

$$4f = \tfrac{1}{2}(325 + 336) = 331$$

[We might adopt this improved value for f to find a truer value for $\bar{s}$, and thence we might get a still nearer approximation to the correct value of f. In practice we find that no greater accuracy would be thus attained.]

Substituting now in equation (A) we have

$$\frac{s_2 - s_1}{2} = 11,210 \,;\; s - \left\{\frac{s_1+s_2}{2} + \tfrac{1}{2}f\right\} = 24,108 - 21,141$$
$$= 2,967$$

$$v_2 = (11,210)^2 - 166 \times 2967$$

and

$$v = 11,188.$$

We have neglected all account of the position of the decimal point up to this stage. It is easy to see that the above value of v means that the cylinder of the chronograph is, at the instant of equality of the lights, rotating at a speed which if uniform would accomplish 1·1188 revolutions per second. At the same instant the cylinder rotated 0·2406 of a revolution in the time taken by the toothed wheel to complete 100 revolutions. Hence the speed of rotation of the toothed wheel is

$$n' = \frac{1{\cdot}1188}{0{\cdot}002406} = 465{\cdot}00 \text{ revolutions per second.}$$

The value of v is calculated by the help of seven-figure logarithms.

Having now studied in detail the method of reduction of this particular observation, the following systematic form for the tabulation of results will easily be understood :—

January 20, 1881, No. 6.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
231 2,642 .. 7,449	2,411 2,404	1,437 5,046	515 9,890 23,072 32,309 45,300 54,433	22,557 (22,419) 22,228 22,124	329 295
Signal at 3,810			Signal at 24,108		
$v'=0\cdot002406$			$\bar{s}=21,141$ $4f=\quad 331$		
$w=\frac{v}{v'}=465\cdot00$			$\bar{v}=\quad 1\cdot1210$ Correction for friction, &c. $=-\quad \cdot0028$ $v=\quad 1\cdot1182$		

Referring to the mathematical theory of our method it will be seen that in order to calculate for each observation the slight correction which must be applied, owing to $\frac{D_A}{D_B}$ being not exactly equal to $\frac{13}{12}$, it is necessary to make a first approximation to the velocity of light neglecting this small correction. Our formula for this first approximation, studying the 12th and 13th equality (which is the best pair for our purpose), is

$$V=\frac{2m(n+n')D_B}{r}$$

The mean value of all our determinations of

$n+n'$ is 880·02	log =	2·9444922
$D_B=3\cdot18845$	„	·5035796
$2m=800$	„	2·9030900
		6·3511618
$r=12$	„	1·0791812
$V=187,060$ first approximation		5·2719806

Also from § 6 of the mathematical theory the first approximation to the value of $2N_B$ is $\frac{n+n'}{r}$

$\log(n+n')$	2·9444922
$\log r$.	1·0791812
$\left\{\begin{array}{l}2N_B=73{,}350\\ \text{first approximation}\end{array}\right.$	1·8653110
$g=1·08181$	·0341510
$\left\{\begin{array}{l}2gN_B=79·3345\\ \text{first approximation}\end{array}\right.$	1·8994620

These are the values which are adopted in calculating the second term in the following reductions.

The rejection of observations.—We resolved never to reject a single observation simply because it differed largely from our average result. The only cases where we rejected an observation were (1) when there was no corresponding observation of the next equality taken at a very short interval of time, and (2) when the traces made by the clock pen on the chronograph showed great irregularities in the second differences, or when they showed that the friction on the axle was abnormally great.

With reference to No. (1) we may state that a quarter of an hour even might alter the value of ρ by fog, &c., and this would invalidate the result.

With reference to No. (2) it is right to say that we generally read off a large number of clock-pen marks corresponding to each observation. From these we could easily judge if the chronograph was working well.

December 21, 1880, No. 1.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
10,051			5,652		
	2,563	11,333	15,553	21,825	
12,614			27,477	21,714	254
	2,573	15,187	37,267	(21,571)	317
17,759			49,048	21,397	
			58,664		
Signal at 13,657			Signal at 33,915		
$v'=\cdot002569$			$\bar{s}=38{,}302$ $4f=\ \ 310$		
$n=\frac{v}{v'}=421\cdot08$			$\bar{v}=\ 1\cdot0786$ Correction for friction, &c. $=+\ \cdot0032$ $v=\ 1\cdot0818$		

December 21, 1880, No. 2.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
5,790			5,370		
	2,316	6,948	17,091	21,385	
8,106			26,755	(21,198)	294
	2,297	10,403	38,289	21,091	185
12,700			47,846	21,013	
			59,302		
Signal at 8,679			Signal at 28,958		
$v'=\cdot002306$			$\bar{s}=27{,}726$ $4f=\ \ 280$		
$n'=\frac{v}{v'}=459\cdot28$			$\bar{v}=\ 1\cdot0599$ Correction for friction, &c. $=-\ \cdot0008$ $v=\ 1\cdot0591$		

From Nos. 1 and 2 $\left\{\begin{array}{r} 2m\dfrac{(n+n')D_B}{r}=187{,}132 \\ 1-\dfrac{\delta}{g+\rho}=0\cdot999323 \\ \text{Product}=V=187{,}007 \end{array}\right\}$ Correction for second term $=-125$.

December 21, 1880, No. 5.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			7,953		
1,197			17,419	16,742	
	1,964	2,179	24,695		294
3,161			..	(16,448)	
..	1,972	5,133	41,143		320
7,104			50,225	16,128	
			57,271		
Signal at 3,482			Signal at 33,692		
$v'=\cdot001968$			$\bar{s}=32,958$ $4f=308$		
$n=\frac{v}{v'}=417\cdot58$			$\bar{v}=\cdot8224$ Correction for friction, &c. $=-\cdot0006$ $v=\cdot8218$		

December 21, 1880, No. 6.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			5,952		
8,448			14,799	19,866	
	2,110	9,503	25,818	19,699	339
10,558			34,498	(19,527)	282
..	2,083	12,641	45,345	19,417	
14,723			53,915		
Signal at 12,241			Signal at 32,524		
$v'=\cdot002086$			$\bar{s}=35,616$ $4f=285$		
$n'=\frac{v}{v'}=469\cdot14$			$\bar{v}=\cdot97640$ Correction for friction, &c. $=+\cdot00224$ $v=\cdot97864$		

From Nos. 5 and 6
$$\left\{\begin{array}{l} 2m\dfrac{(n+n')D_B}{r}=188,484 \\ 1-\dfrac{\delta}{g+\rho}=0\cdot999583 \\ \text{Product}=V=188,405 \end{array}\right\}$$
Correction for second term $=-79$.

December 21, 1880, No. 8.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
7,785 9,628 . . 13,277	1,843 1,825	8,707 11,453	7,927 17,479 25,352 34,756 42,518 51,729	17,425 17,277 (17,166) 16,973	259 304
Signal at 10,263			Signal at 30,460		
$v'=\cdot001833$			$\bar{s}=33{,}972$ $4f=\ 292$		
$n=\frac{v}{v'}=469\cdot88$			$\bar{v}=\ \cdot85830$ Correction for friction, &c. $=+\ \cdot00298$ $v=\ \cdot86128$		

December 21, 1880, No. 9.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences,
5,630 7,936 . . 12,525	2,306 2,295	6,783 10,231	7,000 15,737 26,172 34,790 45,100 53,566	19,172 (19,053) 18,928 18,776	244 277
Signal at 8,924			Signal at 29,096		
$v'=\cdot002299$			$\bar{s}=25{,}295$ $4f=\ 248$		
$n=\frac{v}{v'}=413\cdot32$			$\bar{v}=\ \cdot95270$ Correction for friction, &c. $=-\ \cdot00247$ $v=\ \cdot95023$		

From Nos. 8 and 9
$$\left\{\begin{array}{r} 2m\frac{(n+n')D_B}{r}=187{,}736 \\ 1-\frac{\delta}{g+\rho}=\cdot999678 \\ \text{Product}=V=187{,}676 \end{array}\right\}$$
Correction for second term $=-60$.

December 21, 1880, No. 10.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
3,542 5,795 .. 10,310	2,253 2,258	4,669 8,053	9,604 19,223 30,755 40,272 51,644 61,034	21,151 21,049 (20,889) 20,762	262 287
Signal at 6,918			Signal at 37,186		
$v'=\cdot002256$			$\bar{s}=41,236$ $4f=\ 285$		
$n'=\frac{v}{v'}=464\cdot24$			$\bar{v}=\ 1\cdot0445$ Correction for friction, &c. $=+\ \cdot0028$ $v=\ 1\cdot0473$		

From Nos. 9 and 10
$$\left\{\begin{array}{r} 2m\frac{(n+n')D_B}{r}=186,537 \\ 1=\frac{\delta}{g+\rho}=\cdot999570 \\ \text{Product}=V=186,457 \end{array}\right\}$$
Correction for second term $=-80$.

December 21, 1880, No. 11.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
1,897 4,721 .. 10,373	2,824 2,826	3,309 7,547	2,422 13,039 25,757 36,229 48,770 59,164	23,235 (23,190) 23,013 22,935	322 255
Signal at 5,690			Signal at 25,918		
$v'=\cdot002825$			$\bar{s}=24,675$ $4f=\ 325$		
$n=\frac{v}{v'}=410\cdot12$			$\bar{v}=\ 1\cdot1595$ Correction for friction, &c. $=-\ \cdot0009$ $v=\ 1\cdot1586$		

From Nos. 10 and 11
$$\left\{\begin{array}{r} 2m\frac{(n+n')D_B}{r}=185,857 \\ 1-\frac{\delta}{g+\rho}=\cdot999632 \\ \text{Product}=V=185,788 \end{array}\right\}$$
Correction for second term $=-79$

January 20, 1881, No. 3.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
9,610 12,666 18,767	3,056 3,051	11,138 15,717	6,415 21,264 31,805 46,456 56,885 71,344	25,390 (25,192) 25,080 24,888	310 304
Signal at 13,914			Signal at 34,168		
$v'=\cdot003053$			$\bar{s}=33,899$ $4f=309$		
$n=\frac{v}{v'}=412\cdot53$			$\bar{v}=1\cdot2596$ Correction for friction, &c. $=-\cdot0002$ $v=1\cdot2594$		

January 20, 1881, No. 4.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
2,214 4,960 10,469	2,746 2,755	3,587 7,715	7,554 22,621 33,322 48,207 58,791 73,563	25,768 (25 586) 25,469 25,356	299 230
Signal at 5,291			Signal at 35,490		
$v'=\cdot002750$			$\bar{s}=35,450$ $4f=289$		
$n'=\frac{v}{v'}=465\cdot18$			$\bar{v}=1\cdot2793$ Correction for friction, &c. $0\cdot0$ $v=1\cdot2793$		

From Nos. 3 and 4 $\left\{\begin{array}{r} 2m\frac{(n+n')D_B}{r}=186,569 \\ 1-\frac{\delta}{g+\rho}=\cdot999603 \\ \text{Product}=V=186,495 \end{array}\right\}$ Correction for second term $=-74$.

January 20, 1881, No. 5.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
2,926 5,687 . . 11,243	2,761 2,778	4,307 8,465	8,863 22,476 32,127 45,550 55,022 68,216	23,264 (23,074) 22,895 22,666	369 408
Signal at 6,119			Signal at 36,377		
$v'=\cdot002768$			$\bar{s}=34{,}060$ $4f=\ \ 378$		
$n=\frac{v}{v'}=415\cdot15$			$\bar{v}=\ \ 1\cdot1537$ Correction for friction, &c. $=-\ \ \cdot0046$ $v=\ \ 1\cdot1491$		

January 20, 1881, No. 6.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
231 2,642 . . 7,449	2,411 2,404	1,437 5,046	515 9,890 23,072 32,309 45,300 54,433	22,557 (22,419) 22,228 22,124	329 295
Signal at 3,810			Signal at 24,108		
$v'=\cdot002406$			$\bar{s}=21{,}141$ $4f=\ \ 331$		
$n'=\frac{v}{v'}=465\cdot00$			$\bar{v}=\ \ 1\cdot1210$ Correction for friction, &c. $=-\ \ \cdot0022$ $v=\ \ 1\cdot1188$		

From Nos. 5 and 6 $\left\{\begin{aligned} 2m\frac{(n+n')D_B}{r}&=187{,}088\\ 1-\frac{\delta}{g+\rho}&=\cdot999550\\ \text{Product}=V&=187{,}003\end{aligned}\right\}$ Correction for second term $=-85$.

December 21, 1880, No. 2					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
5,790			5,370		
	2,316	6,948	17,091	21,385	
8,106			26,755	(21,198)	294
	2,297	10,403	38,289	21,091	185
..			47,846	21,013	
12,700			59,302		
Signal at 8,679			Signal at 28,958		
$v'=\cdot002306$			$\bar{s}=27{,}726$ $4f=\ 280$		
$n'=\frac{v}{v'}=459\cdot28$			$\bar{v}=\ 1\cdot0599$ Correction for friction, &c. $=-\ \cdot0008$ $v=\ 1\cdot0591$		

December 21, 1880, No. 3.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			9,432		
8,430			21,958	23,000	
	2,336	9,598	32,432		345
10,766			..	(22,646)	
..	2,317	13,083	55,078		203
15,400			67,330	22,443	
			77,521		
Signal at 12,051			Signal at 42,348		
$v'=\cdot002323$			$\bar{s}=43{,}790$ $4f=\ 282$		
$n''=\frac{v}{v'}=487\cdot89$			$\bar{v}=\ 1\cdot1323$ Correction for friction, &c. $=+\ \cdot0011$ $v=\ 1\cdot1334$		

From Nos. 2 and 3
$$\left\{\begin{array}{r} 2m\,\frac{(n'+n'')D_B}{r+1}=185{,}846 \\ 1-\frac{\delta'}{g+\rho}=1\cdot00185 \\ \text{Product}=V=186{,}190 \end{array}\right\}$$
Correction for second term $=+344$.

January 20, 1881, No. 6.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
231			515		
	2,411	1,437	9,890	22,557	
2,642			23,072	(22,419)	329
	2,404	5,046	32,309	22,228	295
. .			45,300	22,124	
7,449			54,433		
Signal at 2,406			Signal at 24,108		
$v'=\cdot002406$			$\bar{s}=21,141$ $4f=\quad 331$		
$n'=\frac{v}{v'}=465\cdot00$			$\bar{v}=\quad 1\cdot1210$ Correction for friction, &c. $=-\quad \cdot0022$ $v=\quad 1\cdot1188$		

January 20, 1881, No. 7.

Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
6,229			4,722		
	2,381	7,420	18,357	23,272	
8,610			27,994	(23,022)	358
	2,358	10,968	41,379	22,914	306
. .			50,908	22,716	
13,325			64,095		
Signal at 9,144			Signal at 29,346		
$v'=\cdot002370$			$\bar{s}=29,912$ $4f=\quad 352$		
$n''=\frac{v}{v'}=485\cdot87$			$\bar{v}=\quad 1\cdot1511$ Correction for friction, &c. $=+\quad \cdot0004$ $v=\quad 1\cdot1515$		

From Nos. 6 and 7 $\left\{\begin{array}{c} 2m\,\dfrac{(n'+n'')\mathrm{D_B}}{r+1}=186,572 \\ 1-\dfrac{\delta'}{g+\rho}=1\cdot00138 \\ \text{Product}=\mathrm{V}=186,830 \end{array}\right\}$ Correction for second term $=+258$.

January 20, 1881, No. 8.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
			139		
2,823			16,186	27,330	
	2,891	4,269	27,469	(27,140)	264
5,714			43,326	27,066	276
..	2,907	8,621	54,535	26,864	
11,528			70,190		
Signal at 8,014			Signal at 28,248		
$v'=\cdot002905$			$\bar{s}=29,789$ $4f=266$		
$n'=\frac{v}{v'}=467\cdot37$			$\bar{v}=1\cdot3570$ Correction for friction, &c. $=+\cdot0007$ $v=1\cdot3577$		

From Nos. 7 and 8

$$\left\{\begin{array}{r} 2m\frac{(n'+n'')D_B}{r+1}=187,037 \\ 1-\frac{\delta'}{g+\rho}=1\cdot00122 \\ \text{Product}=V=187,266 \end{array}\right\}$$

Correction for second term $=+229$

January 21, 1881, No. 2.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
4,207			4,058		
	2,598	5,506	17,526	24,761	
6,805			28,819		415
..			..	(24,346)	
	2,590	10,690	53,165		394
..			65,962	23,952	
14,574			77,117		
Signal at 9,000			Signal at 39,234		
$v'=\cdot002593$			$\bar{s}=41,043$ $4f=410$		
$n'=\frac{v}{v'}=470\cdot04$			$\bar{v}=1\cdot2173$ Correction for friction, &c. $=+\cdot0015$ $v=1\cdot2188$		

January 21, 1881, No. 3.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
7,503 9,949 .. 14,773	2,446 2,412	8,726 12,361	5,879 16,947 29,678 40,619 53,017 63,829	23,799 (23,672) 23,339 23,210	460 462
Signal at 12,071			Signal at 32,305		
$v'=\cdot002415$			$\bar{s}=28{,}841$ $4f=\ \ 460$		
$n''=\frac{v}{v'}=487\cdot58$			$\bar{v}=\ \ 1\cdot1836$ Correction for friction, &c. $=-\ \ \cdot0061$ $v=\ \ 1\cdot1775$		

From Nos. 2 and 3 $\left\{\begin{array}{r} 2m\frac{(n'+n'')D_B}{r+1}=187{,}897 \\ 1-\frac{\delta'}{g+\rho}=1\cdot00116 \\ \text{Product}=V=188{,}110 \end{array}\right\}$ Correction for second term $=+213$

January 21, 1881, No. 4.					
Wheel.			Clock.		
Reading.	Differences.	At mean reading.	Reading.	Alternate differences.	Alternate second differences.
7,376 9,893 .. 14,859	2,517 2,483	8,635 12,376	4,725 15,768 28,334 39,217 51,650 62,421	23,609 (23,449) 23,316 23,204	293 245
Signal at 11,605			Signal at 31,786		
$v'=\cdot002940$			$\bar{s}=27{,}529$ $4f=\ \ 288$		
$n'=\frac{v}{v'}=469\cdot85$			$\bar{v}=\ \ 1\cdot1725$ Correction for friction, &c. $=-\ \ \cdot0026$ $v=\ \ 1\cdot1699$		

From Nos. 3 and 4 $\left\{\begin{array}{r} 2m\frac{(n'+n'')D_B}{r+1}=187{,}860 \\ 1-\frac{\delta'}{g+\rho}=1\cdot00117 \\ \text{Product}=V=188{,}079 \end{array}\right\}$ Correction for second term $=+219$

The following is a summary of these results :—

12th and 13th equalities.

1880	December 21, Nos.	1 and 2	V=187,707 miles per second.
„	„	5 „ 6	188,405 „
„	„	8 „ 9	187,676 „
„	„	9 „ 10	186,457 „
„	„	10 „ 11	185,788 „
1881	January 20, Nos.	3 „ 4	186,495 „
„	„	5 „ 6	187,003 „
Mean for 12th and 13th equalities :			V=187,076 „

13th and 14th equalities.

1880	December 21, Nos.	2 and 3	V=186,190 miles per second.
1881	January 20, Nos.	6 „ 7	186,830 „
„	„	7 „ 8	187,266 „
„	January 21, Nos.	2 „ 3	188,110 „
„	„	3 „ 4	188,079 „
Mean for 13th and 14th equalities :			V=187,295 „
General mean of both sets . .			V=187,167 „

Multiplying this by the mean refractive index of air (=1·00029) we obtain the value for the velocity in vacuo, viz.: 187,221 miles per second.

This must be corrected for the rate of our clock.

One second of our clock is equal to 0·999723 of a mean solar second.

Dividing the value found for V by this quantity, we obtain the final value for the velocity of the white light from an electric lamp in vacuo, viz. :—

V=187,273 miles per second (log=5·2724757)
=301,382 kiloms. per second (log=5·4791167)
Using STRUVE's constant of aberration 20″·445.
The resulting parallax of the sun is =8″·77.
Distance of the sun =93,223,000 miles.

The value obtained by CORNU,* using the method of FIZEAU, was 300,400 kiloms. per second. He nearly always used the DRUMMOND (or lime) light. A few experiments were made with a petroleum lamp.

* "Annales de l'Observatoire de Paris" (Mémoires, tome xiii.), 1876.

The value obtained by MICHELSON,* using a modification of the method of FOUCAULT, was 299,940 kiloms. per second. He always used the light of the sun when near the horizon (in the early morning or late afternoon), except in a single set of observations where he used the electric light, and which he considered unsatisfactory. Thus we have three series of very carefully conducted experiments to determine the velocity of light, each one differing essentially from the others in their method of research; the results are all very close to each other, and we believe that in the sequel we shall be able to show reasons for the outstanding differences. Grouping the three sets in order we have:—

	Usual source of light.	Method.	Results for V.
			Kiloms.
MICHELSON* . . .	The sun near horizon .	Deflection by mirror	299,940
CORNU†	Lime light	Toothed wheel and eclipses .	300,400
YOUNG and FORBES .	Electric light . . .	Toothed wheel and equalities .	301,382

CORNU—MICHELSON	460	kiloms. per second.	
YOUNG and FORBES—MICHELSON.	1,442	,,	,,
YOUNG and FORBES—CORNU . .	982	,,	,,

After we had completed the observations which have now been reduced, we found reason for believing that the velocity of light depends upon its colour, and further examination of the question confirmed us in this opinion. It seemed useless then to continue to measure the velocity of a light whose colour changes considerably and quite sufficiently to give us values for the velocity of light varying much more than any errors of observation could make them vary. We then devoted our attention to an examination of the question involved in the second part of our research, viz.: whether the velocity of light depends upon its colour, and if so to what degree.

PART II.

RELATIVE VELOCITY OF LIGHTS OF DIFFERENT COLOURS.

Does the velocity of light depend upon its colour?

Before describing the observations which furnish an answer to this question, we will briefly recapitulate the general arrangements in our method of working, so that those who wish it may study this part of our research independently of the rest.

* "Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac." Vol. i., part iii., 1880.

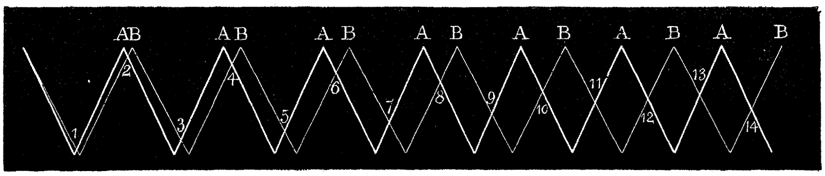
† "Annales de l'Observatoire de Paris" (Mémoires, tome xiii.), 1876.

Our method is very like that of M. FIZEAU. We have an observing telescope, in the focus of which, and in a plane perpendicular to the axis of which, we have a toothed wheel revolving with very great velocity. The light from the sun or from an electric or other lamp is condensed by a lens, and reflected by a diagonal reflector in the eye-piece, so as to throw an image of the incandescent carbons upon the toothed wheel. The light passing between the teeth goes to a distant reflector and returns by reflection to the object-glass of the observing telescope, by which the rays are brought to a focus in the plane of the toothed wheel at that exact part of the focal plane whence the rays had emerged which were capable of striking the distant reflector. This point in the focal plane is of course that point at which the observer sees the distant reflector (through a small aperture in the diagonal reflector), or at which he sees a star of light when the lamp is in action.

If the double distance to the distant reflector and back were about six miles, we know, roughly speaking, light takes about $\frac{1}{31000}$ second to traverse the double distance. Now suppose the wheel to rotate very slowly. We see alternately a tooth of the wheel, and a star of light shining in the interval between two teeth. If the speed be increased so that more than 10 teeth pass in a second, the persistence of visual impressions causes us to be unable to distinguish these alternate phases. We see a star of light continuously upon a partially illuminated field. If in $\frac{1}{31000}$ second a tooth passes into the position previously occupied by a space, then the light which passed away through a space to the distant reflector is on its return stopped by a tooth and we see nothing but the tooth, while when a space between two teeth is at that part of the field of view where the star should appear no star of light is seen, because $\frac{1}{31000}$ second ago a tooth occupied that position, and no light could get through to go to the distant reflector. If there be 400 teeth in the wheel this speed of revolution is $\frac{800}{31000}$ of a second to one revolution, or 38·75 revolutions a second. At this speed of revolution no star of light would be seen, but if the speed be doubled the light passing out through a space to the distant reflector can on its return pass through the next space to the eye of the observer, and the light is seen with its full intensity. If the speed 38·75 revolutions a second be increased threefold, fivefold, &c., or any odd number of times, we have an eclipse of the star. If that speed be increased twofold, fourfold, &c., or any even number of times, we have full brightness.

FIZEAU, and CORNU after him, measured the speed required to produce an eclipse, and thence they deduced a value for the velocity of light. Our method, however, differs distinctly from theirs in this way: that in place of having a single reflector in the distance, we have two at different distances from the observing telescope, but nearly in the same line with it, so that the observer in looking through the telescope sees two stars side by side separated from each other by a distance of about 25″ of arc. While the toothed wheel is being rotated with gradually increasing speed, the star coming from the more distant reflector (which we call A) is eclipsed before that one coming from the nearer reflector (which we call B). As the speed increases A grows brighter, while B is still diminishing in brightness, and at a certain speed they are of equal brightness. In the same way in the different phases, as the speed of the wheel is increased more and more, we have a succession of equalities in the two stars. The distance of A from the observer is to that of B in the ratio of about 13 to 12. Hence it follows that the sixth maximum of B coincides with the seventh minimum of A. In the following figure abscissæ represent speeds of revolution of the toothed wheel, and ordinates intensities of the stars. The intersection of the two lines indicates equality of brightness and shows the speed required to produce it.

Fig. 2.



The lines in the above figure which indicate the brightness are subject to certain alterations dependant upon the necessary imperfections in the optical and mechanical parts of the apparatus. It has been shown in the previous part of this memoir that our method is in general unaffected by these alterations. Most of our observations have been made at speeds corresponding to the 12th, 13th, and 14th equalities, and it will be unnecessary in the diagrams which represent further development of the theory to delineate other parts of the diagram.

Distinctive colours observed in the return light.

Having made these preliminary remarks, we will now proceed to trace the steps by means of which the relation between colour and velocity has been suspected, and the quantity of the effect has been approximately determined.

In the course of our observations made with sunlight at Pitlochry in 1878, and in those made with the electric light at Kelly in 1880–81, we were frequently annoyed by the presence of colour in the stars, one of them appearing reddish and the other bluish. This made it very difficult to appreciate the exact speed which might be said to produce equality in the lights; for, as is well known, it is very difficult, if not impossible, to judge accurately of the equality of two lights of different colours. We considered that these colours arose from a want of accurate adjustment of the distant reflectors. These consist each of a telescope tube with an achromatic object-glass at one end, but with no eye-piece, and having a silver mirror at the focus of the object-glass. An image of the object-glass of the observing telescope is thrown by the rays from the source of light upon the silver mirror, whence the light is reflected back to the observing telescope. Now the quantity of light which is reflected back *into* the observing telescope depends largely upon the accuracy of focus of the reflecting collimator. If, then, the object-glass be not accurately achromatised the one reflector may be focussed accurately for blue rays and the other for red rays. Thus we should have one star intrinsically redder than the other. In consequence of these considerations, we used, whenever we noticed a difference in colour, to mention the fact, in order that this cause of inaccuracy in the observation might be taken account of. We did not take particular notice which star was red and which blue, though we sometimes noted the fact. We had no idea that any information might be gained by always noticing which star was red and which one blue.

On different days the distinctness and steadiness of the stars varied enormously. The days when the stars were steady and distinct were the days on which we got the best observations, and felt most certainty about the exact speeds which produced equality in the two stars, except that it was often on these days that the difference in colour troubled us most.

On the 11th February, 1881, we were making the regular observations for determining the speed of revolution of the toothed wheel required to give the 12th, 13th, and 14th equalities, corresponding to speeds of about 410, 450, and 490 revolutions a second. These speeds were obtained by using three, four, or five weights to drive the mechanism. The observations in the observing book are numbered from 1 upwards. The following observations, 1 to 5, were taken between 9.30 A.M. and 9.50 A.M. The remarks are extracted from the observing book, and were written at the time, an entry being made after each observation.

"*February* 11, 1881.—Splendid morning. A and B very bright and steady. If anything, A is greater than B.

1.—Three weights.	B increasing	with increase	of speed.		B reddish,		A bluish (?).	
2.—Four weights.	A increasing	,,	,,	,,	A	,,	B	,,
3.—Five weights.	B increasing	,,	,,	,,	B	,,	A	,,
4.—Four weights.	A increasing	,,	,,	,,	A	,,	B	,,
5.—Three weights.	B increasing	,,	,,	,,	B	,,	A	,,

(Not a very good observation.)"

The mark (?) expressing doubt about the colour in the first observation was inserted after the second observation had been made, and was so inserted because they seemed to be antagonistic.

When the observations numbered 6 and 7 respectively were being taken at 10.55 A.M., the following remark is entered in the observing book: "The same phenomenon as above." This refers to the colours.

A number of trials were then made at different speeds (in which no use was made of the chronograph) to examine still further this remarkable phenomenon, and the following statement is made in the observing book:—"Always the light which is increasing with respect to the other, with increase of velocity [of the toothed wheel], appears red; and the other one blue." The words in square brackets are not in the original, and are inserted here to make the statement clearer.

These observations clearly proved to us that the colour which we had often observed was not always due to the adjustment of the distant reflectors. For here sometimes the one and sometimes the other was the red one. At each successive equality (*e.g.*, the 11th and 12th, the 12th and 13th, &c.) the colours of A and B are reversed.

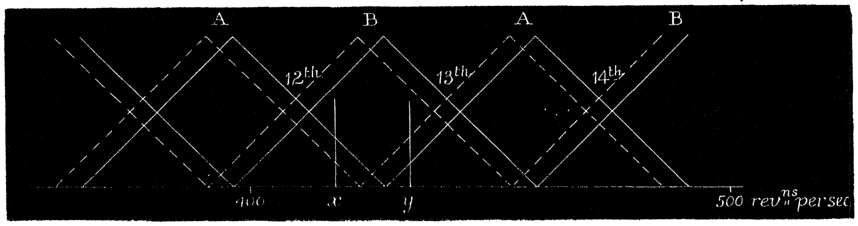
Since February 11 there certainly have been many days when the colour-differences were not perceptible. It may perhaps have been because the stars were not steady or were flickering or indistinct. On these occasions the atmospheric refraction disturbs the course of the rays, so that the teeth of the wheel being extremely minute, a ray of light which, if there were no irregular atmospheric refraction, would not reach the reflector, does so under these circumstances. In such a case the stars do not alter their intensities, with change of speed of the toothed wheel, so regularly as they do when the atmosphere is not unequally heated and disturbed.

The general result however was established by the observations on February 11, 1881, but it is not a common observation.

Explanation of the colours perceived in the return light.

The simplest explanation which can be given of these phenomena, and the only explanation which seems to be capable of standing all kinds of tests, is that *the different colours travel with different velocities, the more refrangible rays, or those with shortest wave-length, travelling quickest.* If this were the case we should be forced to alter our diagram indicating the intensities of A and B. We should have as many curves of intensity for each of the two stars of light as there are colours in the light we are employing. Let us examine only two of these colours (red and blue for example). If the red light travel slower than the blue a smaller velocity is required to produce an eclipse with red light and with blue. For this reason the curve representing intensity in terms of speed of rotation for red light will have its maxima and minima lagging gradually more and more behind those for the blue light. We are in general dealing only with the speeds of rotation which produce the 12th, 13th, and 14th equalities; and during that small variation in speed the lines for red and for blue light may, for purposes of illustration, be drawn sensibly parallel. The curves for the two stars A and B would then be shown approximately by the following diagram, in which dotted lines represent red light, and full lines blue light. Here we notice that at x the light of A is diminishing with increase of speed, and the abscissa corresponding to blue light is greater than that corresponding to red light. Hence, *when the intensity is diminishing with increase of speed the star should have a blue tinge.* But at y the light of A is increasing with increase of speed, and the abscissa corresponding to red light is greater than that corresponding to blue light. Hence, *when the* *intensity is increasing with increase of speed the star should have a red tinge.* Observation confirms these statements. Hence the observations can be explained on the assumption that blue light travels quicker than red light.

Fig. 3.



The analytical expression of this result is quite simple. §§ 2 and 3 of the mathematical theory give the following values for the intensities of red and violet light (indicated by the suffixes R and V):

1. Light increasing with increase of speed
$$\begin{cases} I_R = \frac{E}{2}\left\{2(1-\kappa)-p+\frac{n}{N_R}\right\} \\ I_V = \frac{E}{2}\left\{2(1-\kappa)-p+\frac{n}{N_V}\right\} \end{cases}$$

But if the velocity for violet is greater than for red light, N_V is greater than N_R, and hence I_R is greater than I_V in the even phases, and the return light will appear tinged with red.

2. Light diminishing with increase of speed
$$\begin{cases} I_R = \frac{E}{2}\left\{2(1-\kappa)+p-1-\frac{n}{N_R}\right\} \\ I_V = \frac{E}{2}\left\{2(1-\kappa)+p-1-\frac{n}{N_V}\right\} \end{cases}$$

Here, on the other hand, if N_V be greater than N_R, we have I_V greater than I_R, and the return light will be tinged with blue, in the odd phases.

First measurements of the difference in velocity of red and white light.

While we were quite prepared to examine every possible source of error in these new and unexpected conclusions, we considered it to be of first importance to attempt to get, even in a rough manner, some actual measurements of the difference in velocity of red and blue light, on the assumption that such a difference is the explanation of our results. From the above figure it will be seen that the speed of rotation necessary to give equality of lights must always be greater for blue than for red light. It is also clear that the difference in speed of rotation for red and for blue light bears the same relation to the absolute speed of rotation for either of those colours as the difference in velocity between rays of red and blue light bears to the absolute velocity of that colour.* The same thing will hold, though to a less extent, on comparing red and white light. Our only means at hand on February 11 was to determine the speed which produced an equality (1) in the ordinary way with the white light of the electric lamp, and (2) with the eye screened by a piece of ruby red glass.

The observations made with this object on February 11 are named in the observing book No. 13 (white), No. 13 (red), No. 14 (white), and No. 14 (red).

They were not very satisfactory, for the differences found between the velocities of red and white light were small. The observations No. 13 were at the speed producing the 13th equality. The observations No. 14 were at the speed producing the 14th equality. The speeds of rotation finally deduced from the chronograph records were as follows :—†

		Difference.
Observation No. 13 (red) speed of rotation	$456^{r}\cdot84$	$4^{r}\cdot14$
,, No. 13 (white) ,, ,, ...	$460^{r}\cdot98$	
,, No. 14 (red) speed of rotation	$494^{r}\cdot85$	$1^{r}\cdot57$
,, No. 14 (white) ,, ,, ...	$496^{r}\cdot42$	

$$\frac{\text{Difference of velocity (red and white)}}{\text{Absolute velocity (white)}}=\begin{cases}\cdot90 \text{ per cent. from No. 13}\\ \cdot32 \quad ,, \quad ,, \quad \text{No. 14}\end{cases}$$

These differences are small; but on the whole they indicate a greater speed for white than for red light. But these differences might be suspected to be due to irregularities in the action of the chronograph. The general result seemed to be that we must obtain a greater difference in speed by choosing two colours of light, differing considerably in wave-length, and that we might with advantage discard the chronograph as an absolute measurer of speeds, and adopt some more delicate means of measuring minute differences of speed.

Great difficulty was found in obtaining a blue medium which would sufficiently stop out the red rays. The ordinary blue glass, coloured with cobalt, allows large quantities of red light to pass. We tried eight or nine solutions, which we put into glass cells with parallel sides and tested with a prism. We found that a nitrate of copper solution gave the least quantity of red.

* § 4 of the mathematical theory gives us the value of n for the r^{th} equality for red light, viz.,

$$n_{\mathrm{R}}=\frac{\{2(1-\kappa)(1-\rho)+r(1+\rho)\}\mathrm{N_A N_B}}{\mathrm{N_B}+\rho\mathrm{N_A}}=\kappa.\frac{\mathrm{N_A N_B}}{\mathrm{N_B}+\rho\mathrm{N_A}}$$

If now $\sigma=\frac{\text{velocity of violet light}}{\text{velocity of red light}}$ we notice that for violet light $\mathrm{N_A}$ and $\mathrm{N_B}$ must be multiplied by σ.

Thus we have

$$n_{\mathrm{V}}=\kappa.\sigma.\frac{\mathrm{N_A N_B}}{\mathrm{N_B}+\rho\mathrm{N_A}}=\sigma.n_{\mathrm{R}}$$

$$\frac{n_{\mathrm{V}}}{n_{\mathrm{R}}}=\sigma$$

also $\frac{n_{\mathrm{V}}-n_{\mathrm{R}}}{n_{\mathrm{R}}}=\frac{\text{difference of velocities for red and violet}}{\text{velocity of red light}}$ Q. E. D.

† The observations and reductions are in the hands of the Royal Society.

First differential observations for red and blue light.

After February 11 we never had a clear day or night to continue our research until the 21st of the month. We then made observations in the following manner:—A thick piece of indiarubber tubing was attached to the top of the pulley which supports the weights driving the toothed wheel. At its upper end it was attached to a string, which, passing over a fixed pulley, was held by the observer. The observer adjusted the driving weights so as to be a little in excess of what was required to produce equality of the lights. He then fixed the string tightly, and as the weights descended, the indiarubber was stretched, and diminished the effective driving weight. In this way a beautifully gradual diminution of velocity was obtained, accompanied by as beautifully gradual an increase and decrease in the brightness of the two stars respectively. Our plan was to place the blue solution between the lamp and the diagonal reflector. When equality of lights was attained the observer said "Stop" to the assistant, who then commenced to count seconds on the clock. At the same time the blue solution was replaced by a piece of ruby glass. When equality of red lights was attained the observer again said "Stop." The interval of time was then noted in the observing book. Our intention was to measure by means of the chronograph, at our leisure, the diminution in velocity produced by the action of the indiarubber during a given number of seconds.

Alongside of each observation we entered in the observing book the weights we used. But in the observations of February 21 these were nearly always the same, and it was always the 12th equality which was observed, being a speed of about 400 revolutions a second. It will nevertheless be well here to tabulate the names of the weights employed and their absolute weights, as reference is frequently made in the observing book to them. We usually had one large lead weight to begin with, and this was the one used when one lead weight is spoken of. It weighed 56 lbs. We had two other lead weights which might be added, each weighing 12 lbs. 2 oz. We had five iron weights, each weighing 10 lbs. 7 oz. These were generally supported by an additional iron hook weighing 1 lb. 11 oz. Smaller weights, two of which were called *a* and *b*, each weighing about 1 lb., and others weighing a few ounces, were used as well. The length of the indiarubber tubing which we generally, if not always, used was $10\frac{1}{2}$ inches when unstretched. It lengthens 3 inches with the addition of 1 lb. weight.

With one lead weight (56 lbs.) the fall of the weight when driving the mechanism is 1 inch in 7 seconds.

With three lead weights, one iron weight and hook (92 lbs. 7 oz.), the fall is 1 inch in 5·3 seconds.

[*N.B.*—It must be remembered that the effective weight is half of the actual weight owing to the action of the pulley.]

On February 21 eight observations were made in the manner described above. A little practice was required to get accustomed to the method of observation, but after a few trials the results were wonderfully accordant. The following extract from the observing book contains all the observations made upon that day :—

"*February* 21.—No. 1. At 5.30 P.M. Two lead weights and a. Blue to red—interval $=28$ seconds."

The following were between 7 and 8 P.M. :—

"No. 2.	Two lead weights and a + hook.	Interval $=9\frac{1}{2}$ seconds.	
"No. 3.	Weights as before	Interval $=20$ seconds.	
"No. 4.	,, ,, ...	Interval $=24$ seconds.	
"No. 5.	,, ,, $+b$.	Interval $=45$ seconds.	
"No. 6.	,, ,, ...	Interval $=20\frac{1}{2}$ seconds.	Very good.
"No. 7.	,, ,, ...	Interval $=20$ seconds.	
"No. 8.	,, ,, ...	Interval $=21$ seconds."	

The following remark is added :—

"During the whole of to-day—while trying different methods to detect the difference in velocity of different coloured rays—I have at every step been struck by the enormous difference that exists. It is easy to get a velocity for which A is greater for blue light while B is greater for red light. Sometimes, when A and B are equal for blue, A or B is almost invisible for red, and the other at [near ?] its maximum."

On the evening of February 21 chronograph measurements were made to determine the loss of speed after the indiarubber had been in action for some definite interval of time. The interval adopted was about 18 seconds. The observations and reductions are in the hands of the Royal Society. The results of four such determinations are as follows :—

	No. 1.	No. 2.	No. 3.	No. 4.
Revolutions per second—				
At beginning of observation . .	$487^r{\cdot}22$	$514^r{\cdot}77$	$502^r{\cdot}33$	$502^r{\cdot}33$
At end ,, ,, . .	$480^r{\cdot}28$	$499^r{\cdot}64$	$491^r{\cdot}45$	$496^r{\cdot}60$
Diminution of speed in 18 seconds . .	$6^r{\cdot}94$	$13^r{\cdot}13$	$10^r{\cdot}88$	$4^r{\cdot}73$
,, ,, 1 second . .	$0^r{\cdot}38$	$0^r{\cdot}73$	$0^r{\cdot}60$	$0^r{\cdot}26$

Average . . . . $=0{\cdot}49$ revolution per second.

Although these measurements vary a little amongst each other, they give us sufficiently well a rough knowledge of the rate at which the indiarubber reduces the speed.

The average interval of time between the equality of red and blue lights was 23·5 seconds. This multiplied by 0·49 gives us a difference of 11·5 revolutions a second, or about 2·82 per cent. of the speed producing equality of white light (410 revolutions per second).

It appears then from these experiments that the difference in velocity of red and blue lights is about 2·82 per cent. of the total velocity of light.

The eight separate observations give the following values: 3·36, 1·14, 2·40, 2·88, 5·40, 2·46, 2·40, 2·52.

On February 23 a red solution was substituted for the ruby red glass, as it allowed fewer of the more refrangible rays to pass. In these experiments the chronograph was used directly; the velocity of rotation of the toothed wheel being reduced as before by means of the indiarubber tubing.

The observation book contains the following entry :—

"Lights very bright. A greater than B decidedly. Observations perfectly satisfactory. Interval between first and second signals [those for blue and red lights] about 20 seconds. Whole set [four observations] completed in 10 minutes. Atmosphere very clear all the time.

"No. 1. A increasing with increase of speed. Blue light. (When blue lights were equal, on changing to red light, A was far too bright.)

"No. 2.	A increasing	with increase	of	speed.	Red light.
"No. 3.	B ,,	,,	,,	,,	Blue light.
"No. 4.	B ,,	,,	,,	,,	Red light."

The chronograph was used this day in a different manner to that usually adopted. Contrary to our expectation, the result was not satisfactory, and we do not think that the speeds obtained can be relied upon. We can only say therefore to-day that the general effect was the same as before. The defect we allude to is probably due to a small mistake in counting the number of pen marks in a certain space upon the chronograph cylinder. The readings of the chronograph and the reductions are, however, in the hands of the Royal Society.

Exceptional observation.

On February 24, 10 P.M., the following entry was made :—"Tried two speeds. (1) A increasing with increase of speed, (2) B increasing. Used red and blue lights. There seemed to be a decided tendency for *red* to require a *greater* velocity than *blue*, to produce equality. This is contrary to all our previous experience. A was decidedly brighter than B, but both were tolerably steady and bright."

A few minutes later the following observation was made :—

"With white light.

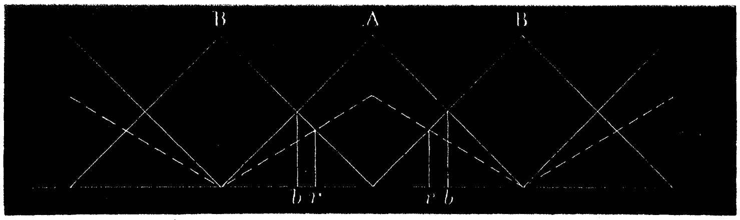
"A blue, B red. Increase of speed increases brightness of A. [This is contrary to previous experience.] This was with speed about 440.

"A blue, B red. Increase of speed increases brightness of B. [This is in accordance with previous experience.] This was with speed about 400.

"It seems then that to-night A is intrinsically bluer than B, owing probably to adjustment of focus. But will this explain all?"

Let us see now what would be the effect of A being intrinsically bluer than B. The following figure represents such a case on the supposition that blue and red light travel with the same velocity, and that A is deficient in red light. Full lines represent blue light, and dotted lines red. These coincide for B.

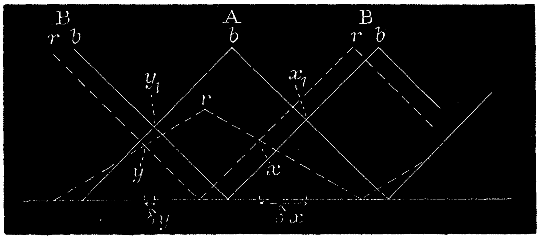
Fig. 4.



The result is clear that at different successive phases the blue and red rays alternately require a greater speed of rotation to produce equality of lights. But this does not in the least represent the results of our observations.

The following diagram illustrates the case where A is deficient in red light, on the assumption that blue rays travel quickest.

Fig. 5.



The maxima of A and B with different speeds are indicated by the letters A, B. The red is shown to reach its maximum before the blue. The red in A is shown to have a smaller maximum than the red in B. The intersections of the full and dotted lines represent the speeds producing equality of the blue and red rays respectively. Here δy is almost nil, whereas δx is too great. Hence, if we observe only at one speed, we may get far too great or far too small a difference between the velocities of red and blue light.

The mean of δy and δx is (nearly) that due to the difference in velocity of red and blue rays.

[*N.B.*—Our observations on February 23 were taken at the 12th and at the 13th equalities, so that this cause of uncertainty is eliminated.]

On February 25, 1881, 4 P.M., B was greater than A. A was bluish compared with B when the toothed wheel was removed. Three observations were made, but they show no decided difference between δx and δy. The indiarubber was used as before.

No. 1. 12th equality of lights	Blue-red = 2·5 seconds.
No. 2. 13th " "	" " = 4·0 "
No. 3. (at 5·30 P.M.) 12th equality of light .	" " = 6·0 "

Now the loss of speed caused by the indiarubber in one second is 0·49 revolution per second. The 12th equality is at a speed of about 420 revolutions a second; and the 13th equality about 460 revolutions a second. From these we get the difference between the velocity for red and for blue light, in percentages of the whole, as follows:—

From No. 1 we have 0·29 per cent.
" No. 2 " 0·43 "
" No. 3 " 0·70 "

On February 27, between noon and 1 P.M., measurements were made in the usual way with the indiarubber, which was a most convenient method of working at different speeds:—

No. 1. 12th equality	Blue-red = 11 seconds.
No. 2. 13th "	" " = $11\frac{1}{2}$ "

(In this observation the indiarubber did not seem to act well after the blue equality).

The observations were continued at 4 P.M. at various speeds and slower than those we had previously experimented upon:—

No. 3. 9th equality	Blue-red = 9·0 seconds.
No. 4. 10th "	" " = 15·5 "
No. 5. 9th "	" " = 16·5 "
No. 6. 10th "	" " = 11·0 "
No. 7. 10th "	" " = 14·0 "
No. 8. 11th "	" " = 9·5 "

The following remarks are appended:—

"Observations from 3 to 8, lights splendidly steady and equal. The difference in velocity for red and blue most striking. A and B are precisely the same colour. Both A and B looked like clearly defined circles with no diffraction phenomena. This is very uncommon, and the observations are therefore valuable."

On the same day some more chronographic measurements were made to determine the reduction in speed per second of time produced by the indiarubber, at the speeds of the 10th and 11th equalities. The reductions are in the hands of the Royal Society, and they gave us—

10th equality . .	0·34 revolutions per second diminution in 1 second.			
11th " . .	0·29	"	"	"
9th " . .	0·31	"	"	"

Reducing the first and second observations as before, allowing for a diminution of speed of 0·49 revolution a second, per second—

$$\text{From No. 1 we have } \frac{\cdot 49 \times 11 \times 100}{420} = 1\cdot 28 \text{ per cent.}$$

$$\text{,, No. 2 ,, } \frac{\cdot 49 \times 11\cdot 5 \times 100}{460} = 1\cdot 22 \text{ ,,}$$

Chronograph records were afterwards taken at about the speeds of the 9th, 10th, and 11th equalities, and the speeds read off at intervals of about 8 or 10 seconds. Thus we obtained the percentage loss of speed in one second. Reducing the observations of this day we have—

$$\text{No. 3. 9th equality; } \frac{\text{difference of velocity (blue and red)}}{\text{total velocity}} = 1\cdot 71 \text{ per cent.}$$

No. 4.	10th	"	"	"	=1·55 "
No. 5.	9th	"	"	"	=3·14 "
No 6.	10th	"	"	"	=1·10 "
No. 7.	10th	"	"	"	=1·40 "
No. 8.	11th	"	"	"	=0·68 "

Mean of the day's observations=1·51

March 1, 1881.—In the interval between February 27 and March 1 we removed the wedge under the toothed wheel and brought the mechanism into an upright position. Seeing that the solutions which we had hitherto used to produce our colours allowed colours of very different refrangibilities to pass, we thought it would be satisfactory to try the effect of pure prismatic colours. To do this we mounted a bisulphide of carbon bottle-prism in front of the condenser and succeeded in throwing a pure spectrum upon the toothed wheel. The distance traversed by the rays from the prism to the toothed wheel was 24½ inches. The electric lamp was mounted upon rollers so that we could easily move it to change the colours. A motion of only about ½-inch of the spectrum changed the colour of the stars from blue to red. This was a very convenient and pleasant method of working. The indiarubber was used as before. The following remarks are extracted from the observing book:—

"Reduced velocity by stretched indiarubber. Used prism for colours. Deflection of spectrum in inches given below [this was measured approximately by our assistant].

"No. 1.	Green to orange. . .	$\frac{1}{2}$-inch,	12·5 seconds,	12th	equality,	good
"No. 2.	Green to reddish-orange	$\frac{5}{8}$-inch,	15 ,,	13th	,,	good
"No. 3.	Blue to orange . . .	$\frac{7}{16}$-inch,	9 ,,	14th	,,	fair
"No. 4.	Deep blue to blood-red	$\frac{5}{8}$-inch,	9 ,,	14th	,,	very good.

"This was a most successful mode of observation. Though B was much greater than A the equality was well determined each time."

In reducing these observations it is clearly impossible to take into account the exact refrangibilities or wave-lengths of the colours named. The general change was from a slightly greenish-blue to a red tinged with orange. Reducing as before, we have—

No. 1. $\frac{\text{Difference in velocity (blue and red)}}{\text{total velocity}} = 1{\cdot}35$ per cent.

No. 2.	,,	,,	$=1{\cdot}56$	,,
No. 3.	,,	,,	$=0{\cdot}90$	,,
No. 4.	,,	,,	$=0{\cdot}90$	,,

March 8, 1 P.M.—It might, perhaps, be thought that we had now thoroughly tested and confirmed our first conclusions by the variety of our tests. But we were anxious to leave no room for doubt and to vary the tests in every possible way, and we thought that some advantage might be gained by changing entirely our method of altering the speed, so as gradually to increase it, the reverse of what we had done with the indiarubber. Accordingly we attached an iron crucible to the weights and led a thick indiarubber tube from it up to a funnel with a stop-cock, in reach of the observer. We filled this with mercury, and having put on such weights as were barely sufficient to produce equality, we turned on red light (using the prism method) and opened the stop-cock. So soon as equality of red lights was attained, we said "Stop" to the assistant, who then commenced to count seconds on the clock. Meantime he had instantaneously changed the colour of the light to blue. When the blue equality arrived we again said "Stop." He gave us the interval in seconds, and the deflection of the spectrum in fractions of an inch, which we entered in the observing book. Chronograph tests were afterwards made to measure the increase in speed produced in a second by the flow of mercury. This method seemed to us to be hardly of so great delicacy as the indiarubber method, but we felt much interest in seeing whether so entirely a different method could give results approximating to those obtained in the previous researches.

The following remarks are extracted from the observing book :—

"*March* 8, 1 P.M.—No. 1. Blue-red =16 seconds. Exact time uncertain to a few seconds. General effect positive" [*i.e.*, the blue equality required a higher speed].

"After blue equality turned again to red, and B was much brighter" [than A], "B increasing with increase of speed. Spectrum deflection about $\frac{5}{8}$ inch." [This was the 12th equality.]

"No. 2. Ditto. Same in every way. Blue-red =9 seconds. $\frac{5}{8}$ inch deflection.

"No. 3. Same arrangements. Blue-red =5 seconds nearly. Deflection =$\frac{3}{4}$ inch.

"Want of oil in (1), (2), and (3) made velocity irregular. We tried one more observation, but the motion of the wheel seemed quite irregular, and though it gave the same value (16 seconds) as No. 1, we could not trust it, because on turning on red light again A was brightest. We now oiled the mechanism with great effect in steadying the motion and diminishing noise. But the lights had begun to flicker and we got no good observations."

At 4 P.M. the lights improved.

"4 P.M.—Oiled axles.

"No. 4. 12th equality. Blue-red =8 seconds. Deflection =$\frac{9}{16}$ inch; but it did not look so great a change of colour as usual.

"No. 5. Same arrangements. Blue-red =10·5 seconds. Deflection =$\frac{11}{16}$ inch. (The change in colour looked greater.) After equality of blue, tried red again, B was far brightest.

"No. 6. Same arrangements. Blue-red =6 seconds. Deflection =$\frac{11}{16}$ inch.

"No. 7. 11th equality. Blue-red =7 seconds. Deflection =$\frac{5}{8}$ inch.

"No. 8. Ditto. Blue-red =6·5 seconds. Deflection =$\frac{5}{8}$ inch."

"8.45 P.M.—No. 9. Same conditions as No. 1" [12th equality]. "Blue-red =16 seconds. Deflection =$\frac{5}{8}$ inch.

"No. 10. Same conditions. Blue-red =23 seconds. Deflection =$\frac{5}{8}$ inch. Speed varied very slowly.

"Oiled the mechanism.

"No. 11. Blue-red =10 seconds. Deflection =$\frac{1}{2}$ inch [12th equality].

"No. 12. Blue-red =9·5 seconds. Deflection =$\frac{5}{8}$ inch [12th equality].

"No. 13. Blue-red =9 seconds. Deflection =$\frac{3}{4}$ inch [11th equality].

"No. 14. Blue-red =7 seconds (good). Deflection =$\frac{3}{4}$ inch [11th equality]."

Reducing these results with the aid of the chronograph tests for the mercury method, we obtain the following 14 percentages for the values for

$$\frac{\text{difference of velocity of blue and red light}}{\text{mean velocity of light}}:—$$

(1) 3·20	(4) 1·60	(7) 1·40	(10) 4·60	(13) 1·80
(2) 1·80	(5) 2·10	(8) 1·30	(11) 2·00	(14) 1·40
(3) 1·00	(6) 1·20	(9) 3·20	(12) 1·90	

The mean of these is 2·03 per cent., giving a result which confirms in a satisfactory manner the results arrived at by the other methods.

April 26 and 27.—After an absence of a month from Kelly we again tested the colour effect on April 26 and 27. We used wheels with 250 teeth and with 400 teeth, bright and smoked, upright and inclined. We ascertained the fact that blue light required the greater velocity of rotation to produce equality; but we did not make any actual measurements. We were unable to detect the successive phases of colour which were observed on February 11.

By collecting results of all the observations on the effect of colour on velocity we shall be able to judge of the general effect:—

February 11.—The star which was increasing with increase of speed was red; the star which was waning with increase of speed was blue.

The percentage difference of red and white light, two observations, 0·90 and 0·32.

February 21.—Ruby-red glass and blue copper solution. Speed diminished gradually by indiarubber. Eight observations—

(1) 3·36	(3) 2·40	(5) 5·40	(7) 2·40
(2) 1·14	(4) 2·88	(6) 2·46	(8) 2·52

Mean percentage difference =2·82.

February 23.—Red solution and blue solution. Direct measurements of the change of velocity. Two observations at different speeds—

(1) 1·96 (2) 0·36

Mean percentage difference =1·16.

February 24.—The effect seemed to be of the opposite kind to that hitherto obtained.

February 25.—Red and blue solutions. Indiarubber method. Three observations—

(1) 0·29 (2) 0·43 (3) 0·70

Mean percentage difference =0·47.

February 27.—Red and blue solutions. Indiarubber method. Eight observations—

(1) 1·28	(3) 1·71	(5) 3·14	(7) 1·40
(2) 1·22	(4) 1·55	(6) 1·10	(8) 0·68

Mean percentage difference =1·51.

March 1.—Pure spectrum thrown by bisulphide of carbon prism upon the toothed wheel. Blue and red parts of the spectrum used in succession. Indiarubber method. Four observations—

(1) 1·35 (2) 1·56 (3) 0·90 (4) 0·90

Mean percentage difference =1·17.

March 8.—Spectrum used for colour. Mercury used to increase speed gradually. Fourteen observations—

(1) 3·20	(4) 1·60	(7) 1·40	(10) 4·60	(13) 1·80
(2) 1·80	(5) 2·10	(8) 1·30	(11) 2·00	(14) 1·40
(3) 1·00	(6) 1·20	(9) 3·20	(12) 1·90	

Mean percentage =2·03.

TABULAR statement of percentage differences in the velocity of red and of blue light.

February 21.	February 23.	February 24.	February 25.	February 27.	March 1.	March 8.	April 26.	April 27.
+3·36	Positive effect.	— . . .	+0·29	+1·28	+1·35	+3·20	Positive effect.	Positive effect.
1·14			0·43	1·22	1·56	1·80		
2·40			0·70	1·71	0·90	1·00		
2·88				1·55	0·90	1·60		
5·40				3·14		2·10		
2·46				1·10		1·20		
2·40				1·40		1·40		
2·52				0·68		1·30		
						3·20		
						4·60		
						2·00		
						1·90		
						1·80		
						1·40		
Mean +2·82	+ . . .	— . . .	+0·47	+1·51	+1·17	+2·03	+ . . .	+ . . .

General mean of 37 observations =1·8.

We cannot account for the apparently negative effect obtained on February 24. But when we consider that it was only for a short time that the observations were attempted, and that the appearance of a negative effect was extremely feeble (almost doubtful), and that in so delicate an observation the state of our health might affect the sensitiveness of our appreciation of colour effects, and that on other days the positive effect was most marked and indubitable, as recorded in numerous passages in our observing book, and quoted above,—when we consider all these things we cannot place this single dubious night's evidence in opposition to the overwhelming testimony of so many positive observations. It was too much to expect that we should be able in the first instance to measure, by the differential methods described above, the difference produced by a given difference of wave-length with very great precision. But there seems little room for doubt that the general mean of these 37 observations cannot be far from the truth, and we may affirm that when the wave-length changes from about $\lambda=50$ tenth-metres to about $\lambda=60$ tenth-metres, the velocity changes about 1·8 per cent., or in any case somewhat over 1 per cent.

Let us now consider some of the effects which must follow from these results.

Refraction and dispersion.—There is nothing in the undulatory theory of light, independent of our views as to the kind of force acting in the ether, which is opposed to the view that red and blue lights travel with different velocities in vacuo. We know that the ratios of the velocities of red or blue lights, in a refracting medium and in vacuo, are equal to their refractive indices. But the theory of refraction or dispersion tells us nothing about the ratio of the velocities of red and blue lights either in vacuo or in refracting media.

Interference and diffraction.—So with the phenomena of interference and diffraction, which give us a measure of the wave-length of the different colours. If we could measure the period of vibration of the different colours, this would show us the difference in velocity in the different colours. But our knowledge of the period of vibration is dependent on the velocity of light, and these phenomena give us no information upon the phenomena we are discussing.

Experiments of CORNU *and* MICHELSON.—Why did not these experimenters notice the effect of colour? First, consider the work of CORNU. We might expect that he should have seen colours in his star of light near the eclipses. But when we have a very feeble light changing its intensity it is very difficult to appreciate differences in colour, and he had no light for comparison as we had. He depended largely for the accuracy of his results on two facts: (1) that he took a mean of 546 pairs of observations, so reducing the probable error to one twenty-third part of that of a single pair of observations, and (2) that he used eclipses as high as the 21st eclipse, which reduces the personal error of the eye observation to one forty-first part. We may add that he had five phases (which he calls U, u, V, v, and ν) differing slightly, and he had considerable latitude in placing any single observation in one or other of these phases. We may also add that into each set of observations he introduced what he called "rectifications probables," which assisted to eliminate discordances.

If, however, we examine his non-rectified results, and classify them according to the source of light used, we shall, we believe, obtain confirmation of our views.

Neglecting the small correction depending upon the phase ($U+u$ or $V+v$) we have the following results:—

Source of light.	Number of observations.	Approximate velocity.
		Kiloms.
Petroleum	20	298,776
Sun near horizon . .	77	300,242
Lime light	449	300,290

Compare this with the absolute determinations given on p. 270.

	Usual source of light.	Velocity.
MICHELSON	Sun near horizon. .	299,940
CORNU	Lime light	300,400
YOUNG and FORBES .	Electric light . . .	301,382

In every case the more refrangible the mean colour of the source of light the greater is the velocity.

In MICHELSON'S observations the image of the slit was described as indistinct and covering a sensible space. From our results it would appear that the width of his spectrum between mean red and blue would be about 2 millims. But it would be a very impure spectrum, and it is only by employing absorptive media, or part of a pure spectrum, to give colour to the light used, that we should expect him to detect the difference.

Solar eclipses and occultations.—We might expect that the sun or a star on disappearing behind the moon would appear to be red, and on reappearing would first flash out blue. But light takes only $1\frac{1}{2}$ second to come from the moon and the one-fiftieth or one-hundredth part of this is too small an interval of time for us to have any hopes in this direction.

Eclipses of Jupiter's satellites.—There must be a difference in time between the disappearance or reappearance of the blue or red parts of the light of a satellite, amounting to about half a minute. But the change is so gradual and the light when near eclipse so feeble that we fear this would be a very difficult determination.

Aberration.—A star exposed fully to the action of aberration must be drawn out into a spectrum parallel to the direction of the earth's motion. The length of the spectrum between the mean red and blue must be about $0''\cdot36$. It is possible that with a reflector of good definition this effect might be detected.

Temporary stars.—The mean light from the nearest star whose parallax is known takes $3\frac{1}{2}$ years to reach the earth. The difference in time taken by the blue and red rays to reach us must be 13 days. The star T Corona flashed out suddenly in 1866. It was first seen on May 9, and there seems to be good evidence to show that it was not conspicuous a weak earlier. On May 9 it was of the second magnitude, and it diminished in brightness for some days at the rate of a magnitude a day. On the 12th May it was examined by Dr. HUGGINS with a spectroscope and showed bright lines in very different parts of the spectrum. This is somewhat contrary to what we should have expected. But we are not aware that the evidence for the identity of the temporary star and the small one known to be in that position is irresistible, nor do we know that the parallax of that small star has been very carefully studied, nor is the evidence as to the invisibility of the star at the beginning of the month conclusive.

Variable stars.—When a variable star brightens the first colour to reach us in increased intensity should be blue, and when it fades the last colour to cease shining in full splendour should be red. If we could determine the difference in time between the time of maximum for blue and for red rays, the present research would enable us to determine approximately the distance of the star from us.

Variable stars are certainly known to change colour; but the only reference to a systematic law in this connexion which we have ever come upon is in WEBB'S 'Celestial Objects for Common Telescopes,' where (Edition II. (1866), p. 208) the following remarkable passage occurs :—

"HIND thinks several variable stars increase blue, are yellow after maximum, and flash red in decreasing."

This is in exact accordance with the results of the present research.

We have applied to Mr. HIND for details of his observations. He informs us that the remark quoted by WEBB was made in a letter to the "Times," some twenty-six years since, on the occasion of notifying a newly discovered variable. He does not, however, feel justified in now advising us to place reliance on this as a law.

It appears, then, that our conclusions as to the relation between colour and velocity must for the present rest upon the merit of our observations. We have ourselves no hesitation in saying that the effects were so striking and so decided on the greatest number of occasions, that we ourselves have no doubt as to the general conclusion. But we admit that much is still left to be done in the way of absolute determinations. There is little doubt that a further investigation into the matter would help to a further knowledge of the properties of the ether (if there be such a substance). This end would be specially aided by an exact determination of the law of dependence of velocity upon wave-length. We believe that, with certain improvements in our apparatus, in a better climate, this determination might be completed.*

* The details of our observations on the dependence of velocity on colour are deposited with the Royal Society.